where Mathematics meets Biology

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An interesting series of numbers

F: 1, 1, 2, 3, 5, 8, 13, 21, 34, ... (value)

n: (1) (2) (3) (4) (5) (6) (7) (8) (9) (index) (plural is *indices*)

- Can you guess the next number?
- Each number is the sum of previous two:

F(1) = 1

F(2) = 1

F(3) = F(2) + F(1) = 1 + 1 = 2

F(4) = F(3) + F(2) = 2 + 1 = 3

F(5) = F(4) + F(3) = 3 + 2 = 5

Some cute properties of the series

F: 1, 1, 2, 3, 5, 8, 13, 21, 34, ... (value)

n: (1) (2) (3) (4) (5) (6) (7) (8) (9) (index)

- Every third value starting from F(3) is always even

even + odd = odd

odd + odd = even

Some cute properties of the series

F: 1, 1, 2, 3, 5, 8, 13, 21, 34, . . . (value)

n: (1) (2) (3) (4) (5) (6) (7) (8) (9) (index)

- Every third value starting from F(3) is always even
- F(1) + F(3) + F(5) + ... + F(2n – 1) = F(2n)

$n = 4$

$2n - 1 = 7$ and $2n = 8$

$$F(1) + F(3) + F(5) + F(7) \stackrel{?}{=} F(8)$$

Some cute properties of the series

F: 1,1,2,3,5,8,13,21,34 −1,... (value)

n: (1) (2) (3) (4) (5) (6) (7) (8) (9) (index)

- Every third value starting from F(3) is always even
- $F(1) + F(3) + F(5) + \ldots + F(2n - 1) = F(2n)$
- $F(2) + F(4) + F(6) + \ldots + F(2n) = F(2n + 1) - 1$

$n = 4$

$2n = 8$ and $2n + 1 = 9$

$$F(2) + F(4) + F(6) + F(8) \stackrel{?}{=} F(9) - 1$$

Fibonacci series

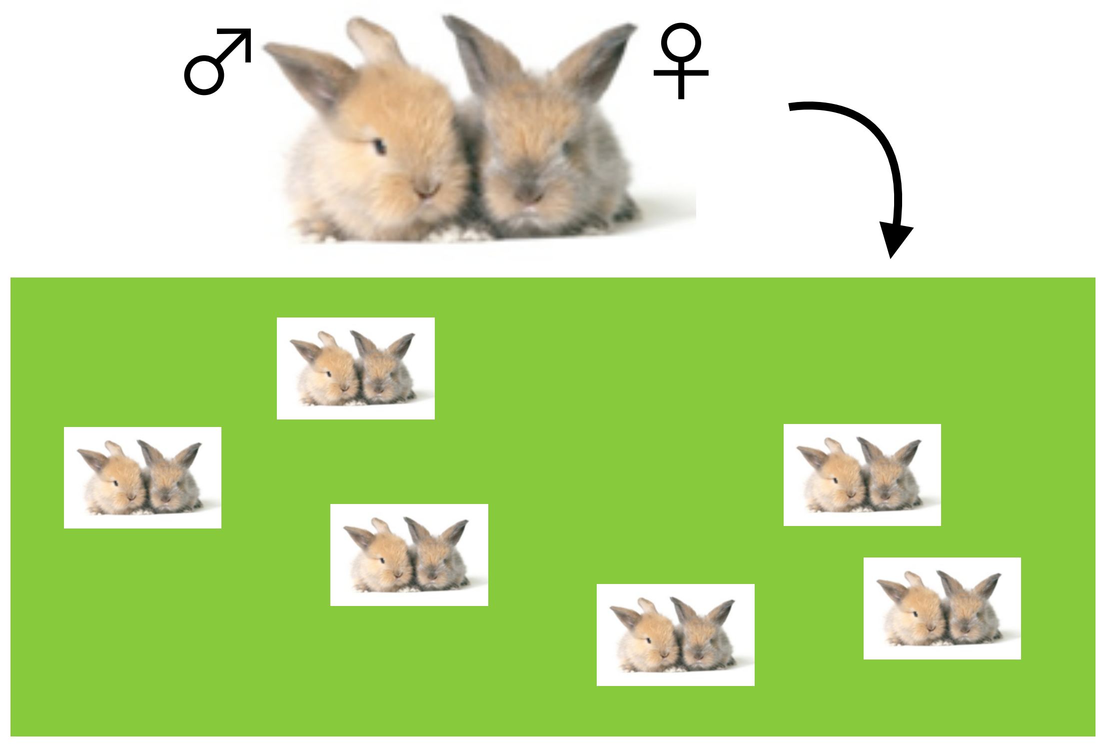
Leonardo Fibonacci

1170 - 1250

Developed this series to solve a ***biological*** problem

Predict the rabbit population

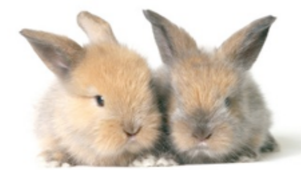
- Suppose a newly born pair of rabbits (a male and a female) are put in a field



How many rabbits will there be after one year?

What assumptions?

♂ ♀



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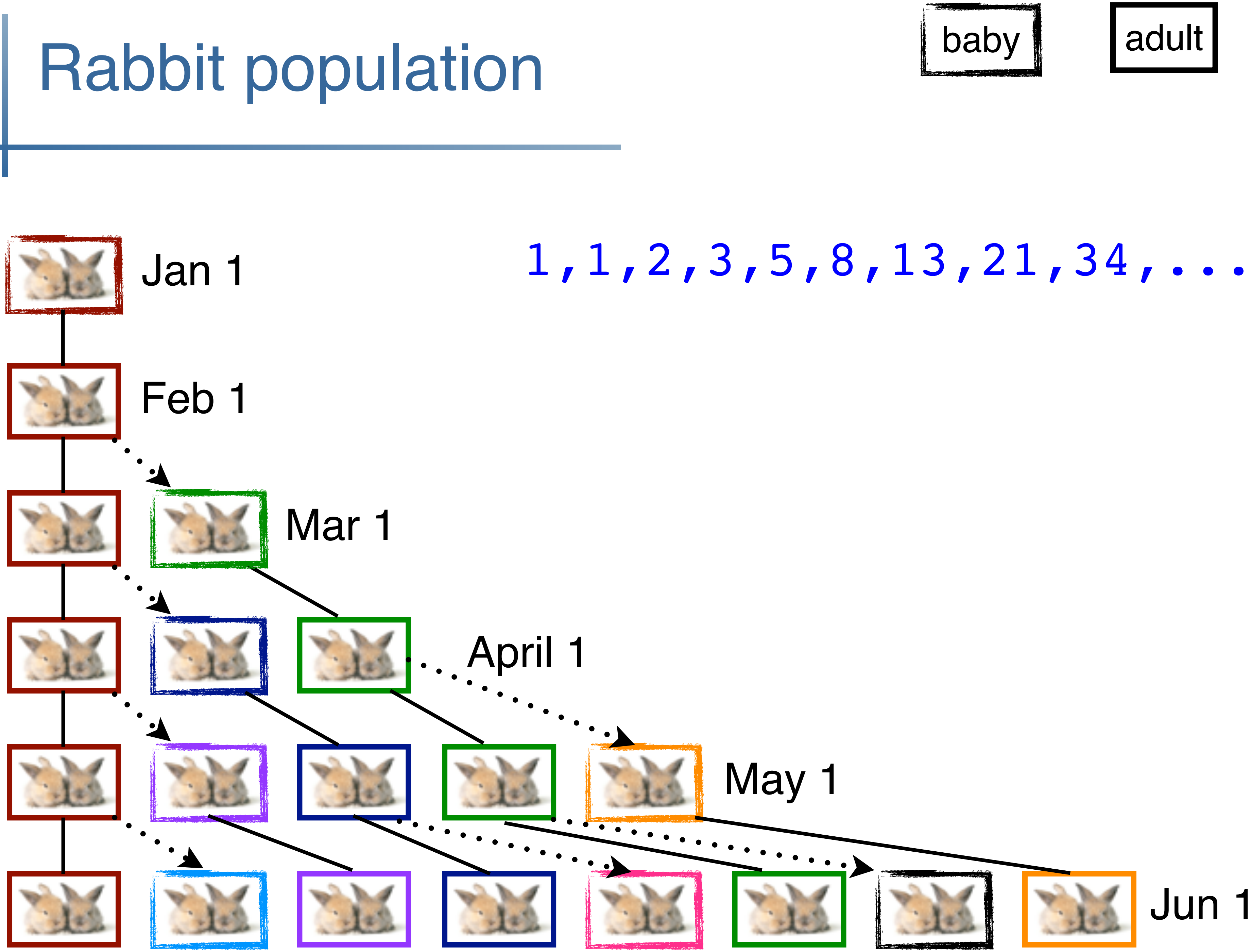
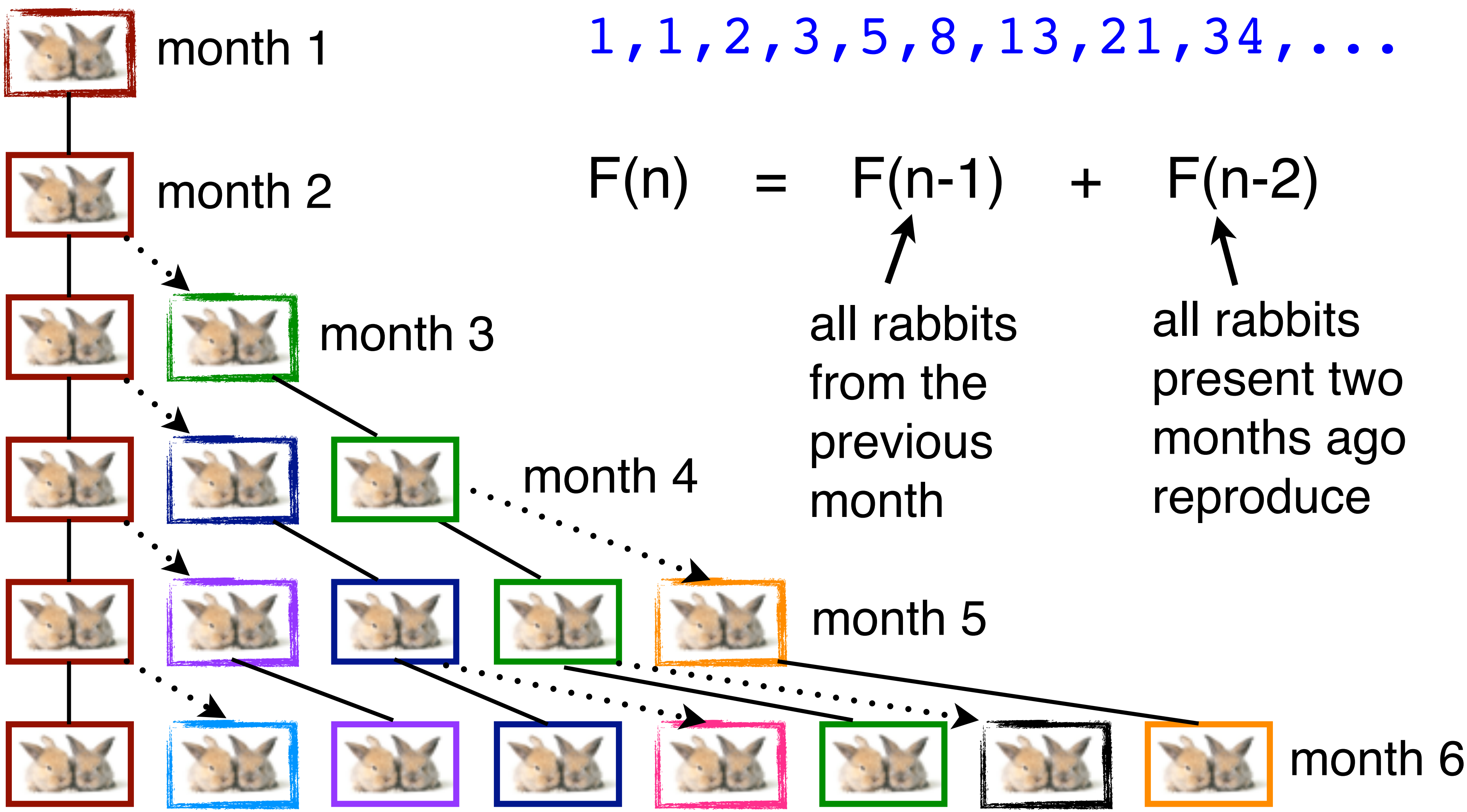
- Rabbits take one month to mature
- After that they produce a new pair of rabbits every month
- Each new pair is always one male and one female
- No rabbit ever dies

Rabbit population



Rabbit population



1,1,2,3,5,8,13,21,34,...

$$F(n) = F(n-1) + F(n-2)$$

$F(n-1)$: all rabbits from the previous month

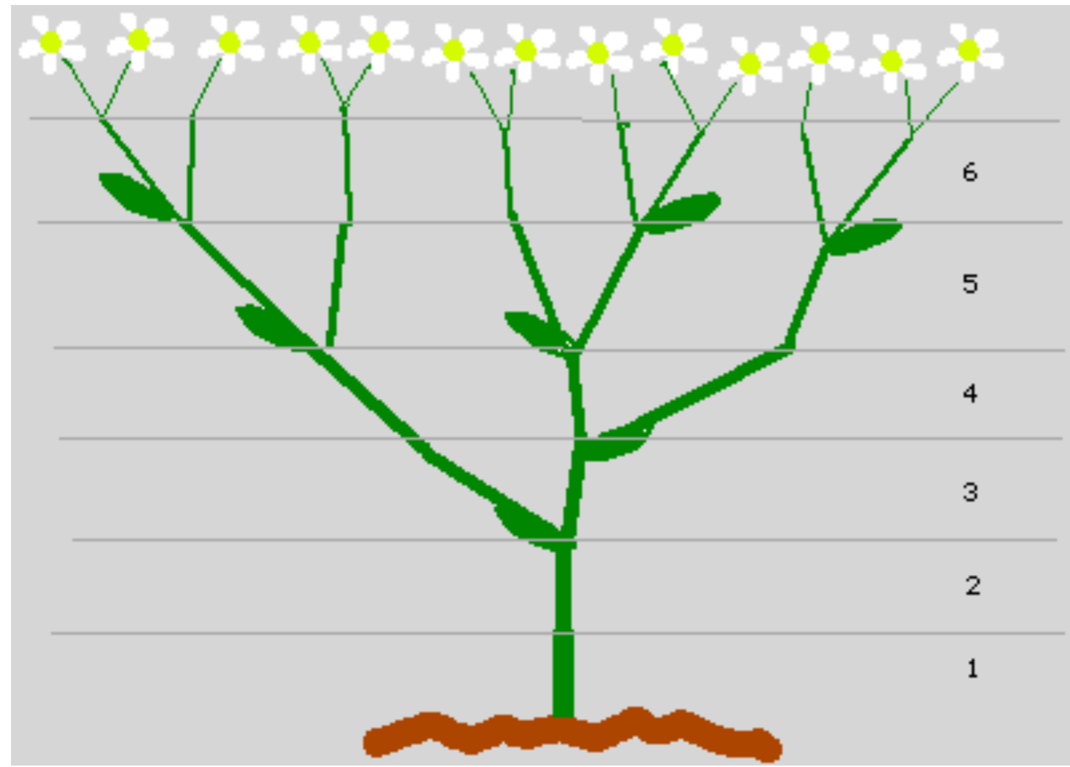
$F(n-2)$: all rabbits present two months ago reproduce

Rabbit population

- This model makes many unrealistic assumptions!
  - rabbits don't die
  - females always give birth to a male & a female
  - they necessarily reproduce every month

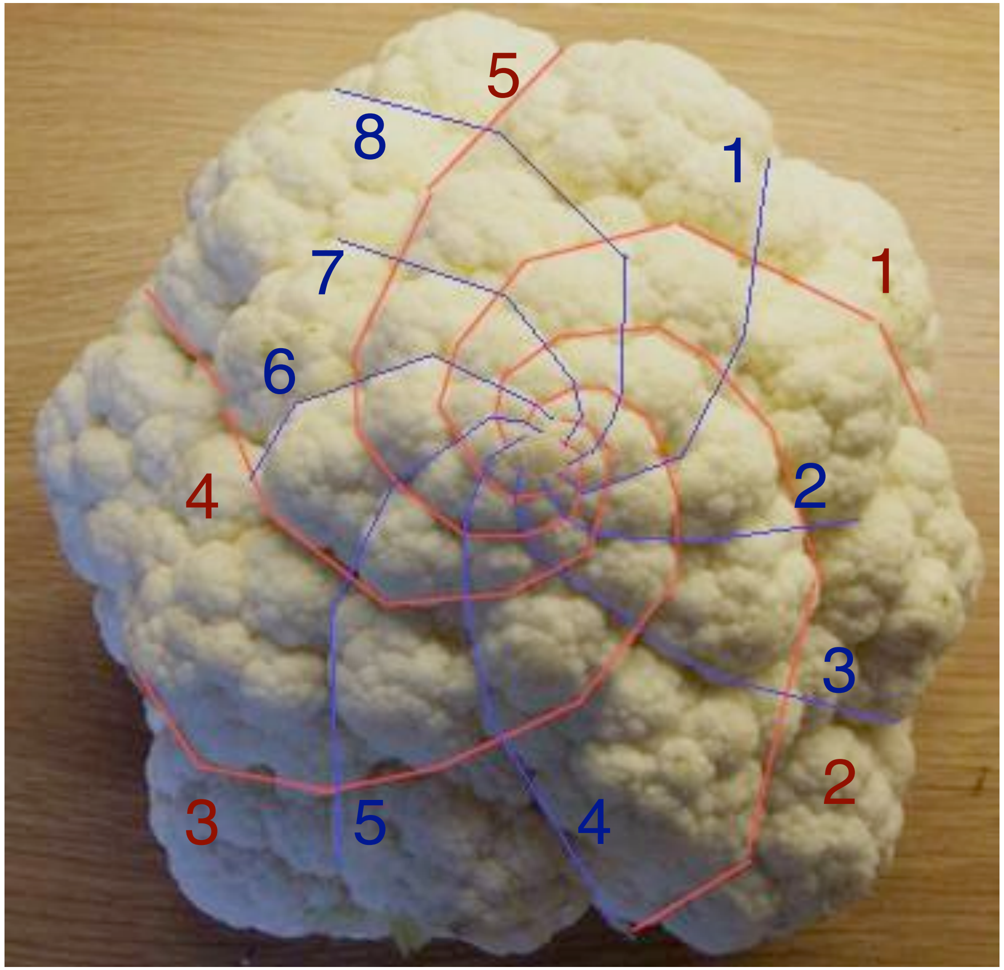
But we do see Fibonacci series in nature

- Some plants display it: when it pulls out a new shoot, it has to grow for sometime (let's say) 2 months before it is strong enough for branching
- New shoot branches every month



http://www.maths.surrey.ac.uk

Anti-clockwise spirals in cauliflower



1, 1, 2, 3, **5**, **8**, 13, 21, 34, ...

Back to rabbits: Fibonacci is not realistic

- What needs to be added to the model?
  - chance of rabbits dying naturally
  - chance of female giving birth to more/less pups
  - chance of another herbivore competing for carrots
  - chance of getting eaten by a predator
  - chance of contracting a disease
  - chance of a natural calamity - droughts, earthquake

How do we model “chance”?

- Probability of an event is a number between 0 and 1 that reflects your “belief” in the event happening

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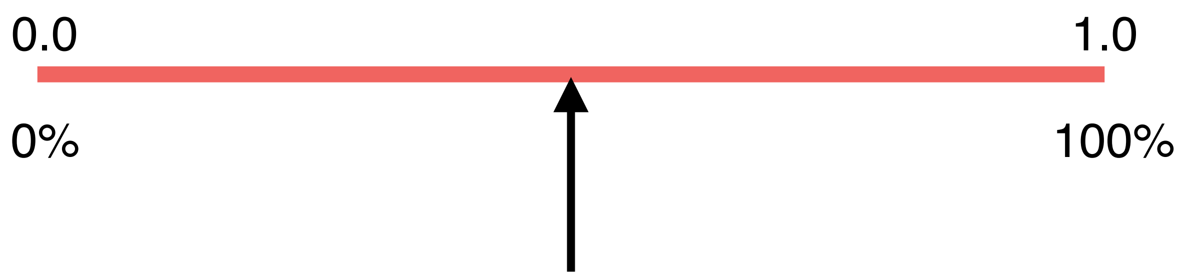
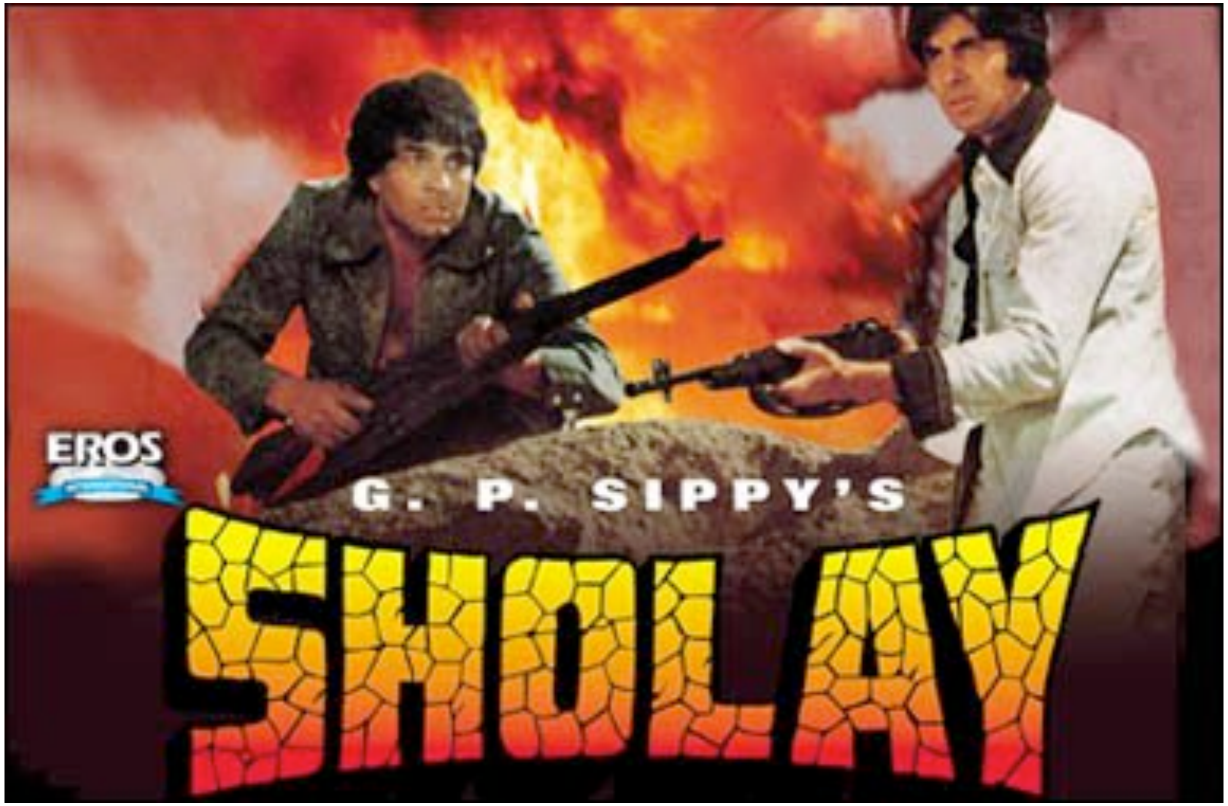


- Suppose you toss a coin, what is the probability that you will get a Head?

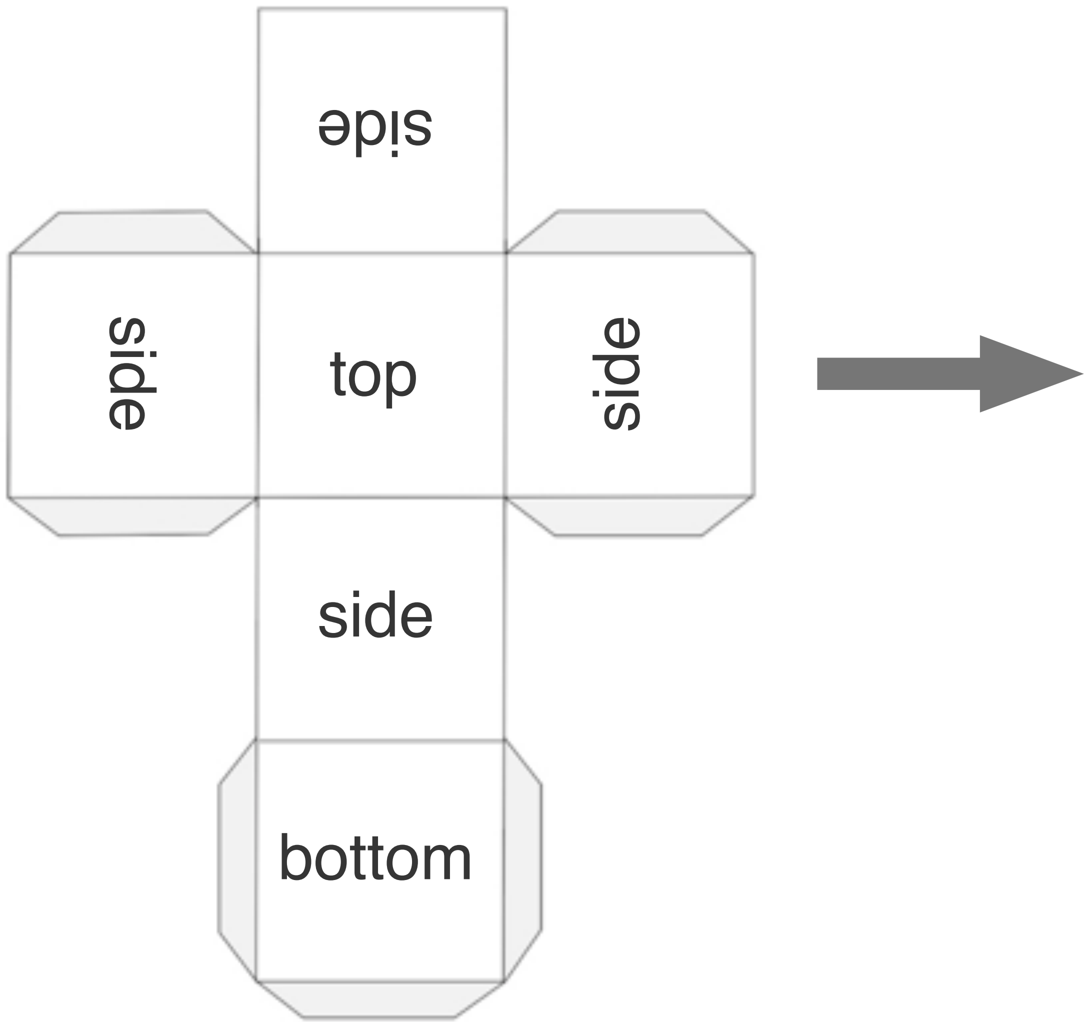
What if it is a Sholay coin?

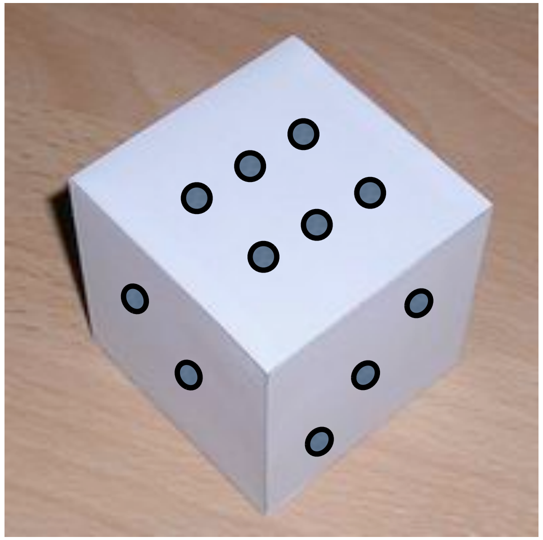


How do you estimate the probability?

- Toss the coin 100 times, count the Heads
- Toss it 1000 times, 10000 times...
- You estimate the *parameters* from data
- How about dice?

Make your own die

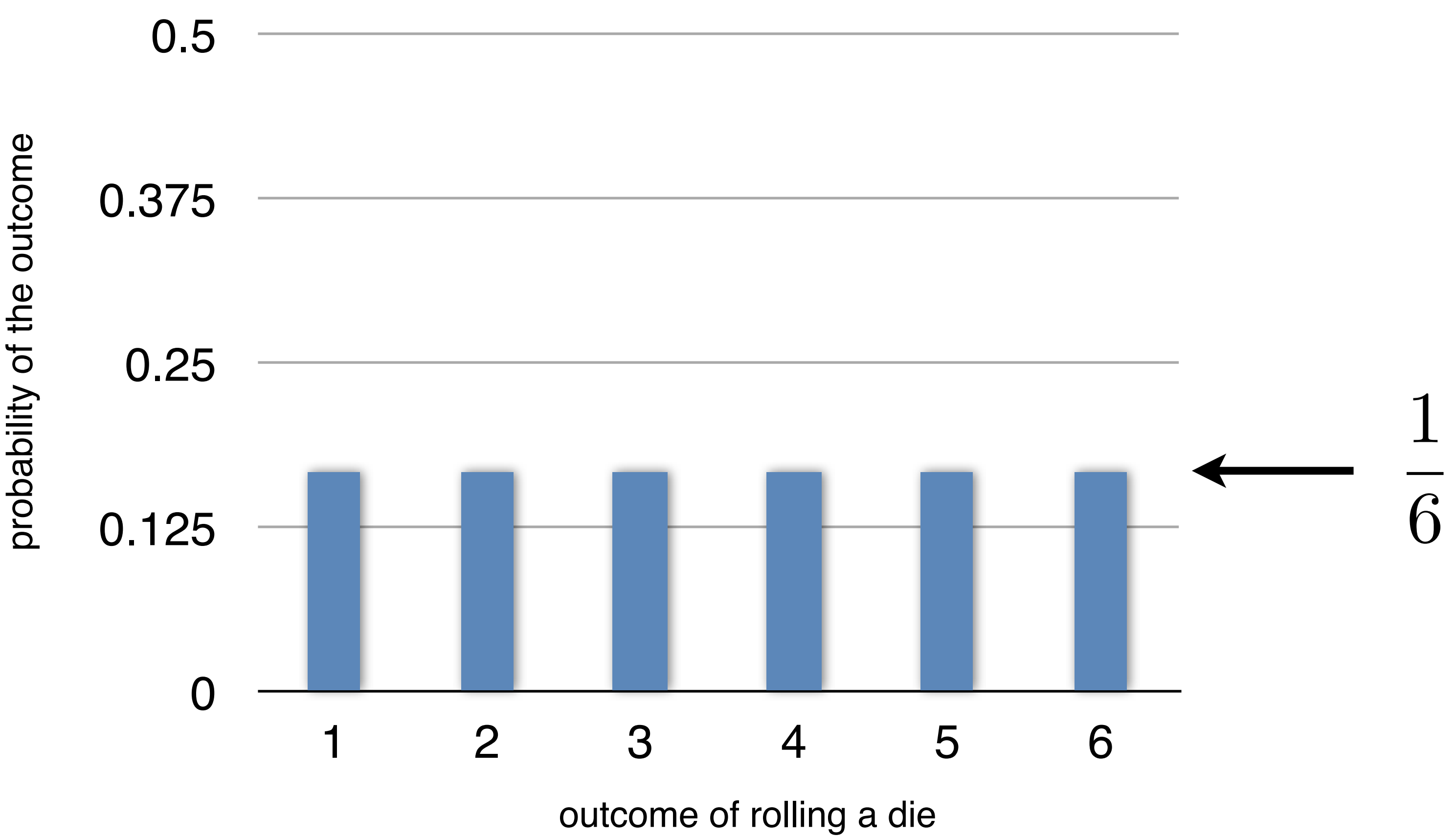




Guessing game for a fair die

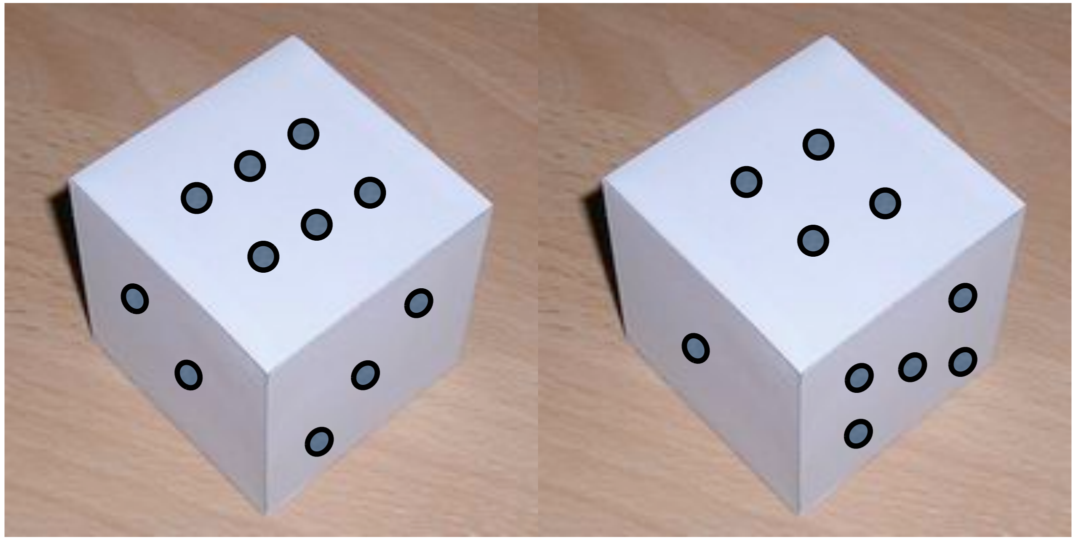
- Your friend rolls a die, you have to guess what number will turn up
- All numbers 1 to 6 in a fair die are equally likely
- You can pick any number...
  - you will be right around 1 in 6 times

Graphically speaking...



What about ***two*** fair dice?

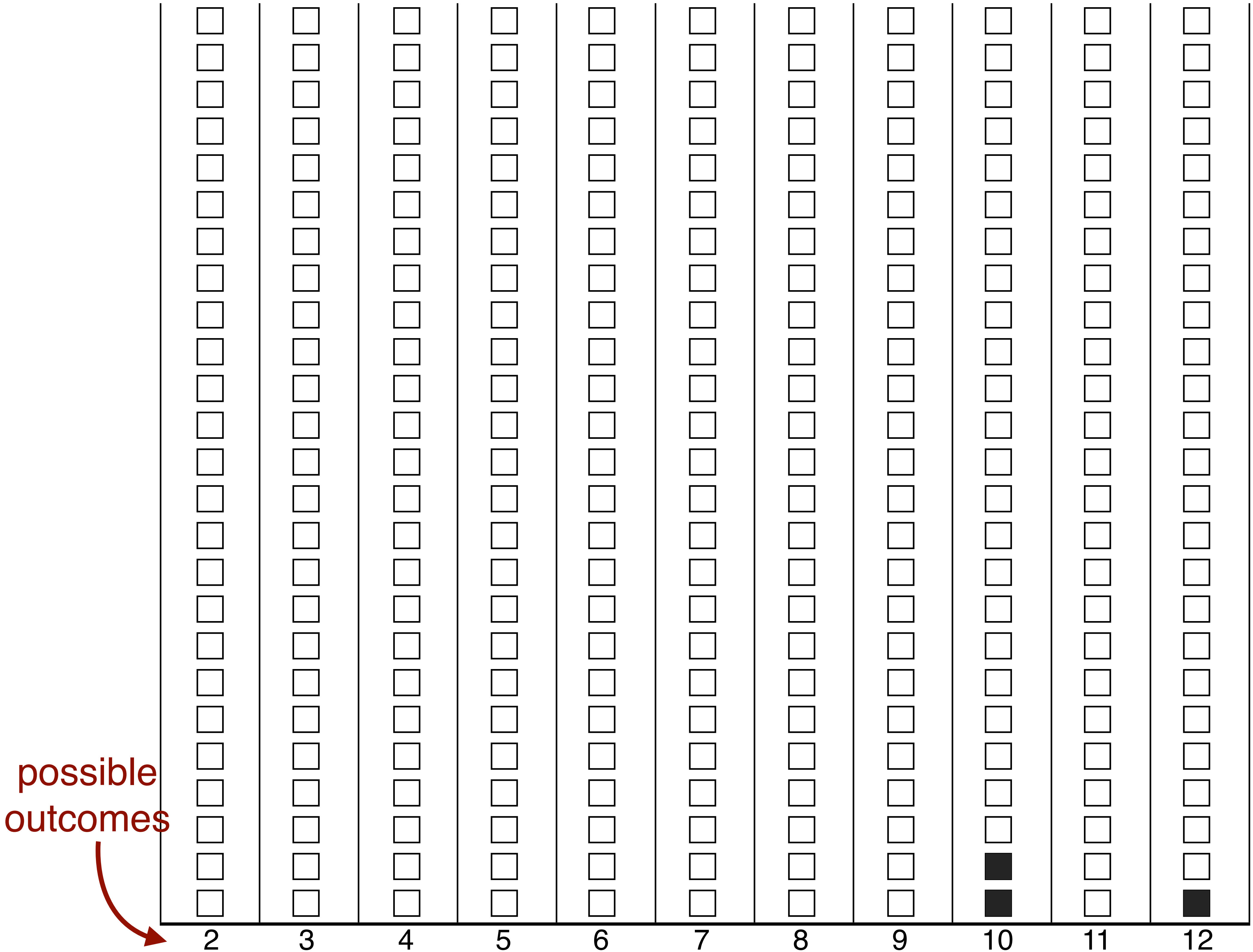
- Outcome of two dice = sum of individuals
- All 1-6 numbers are equally likely for *each* die



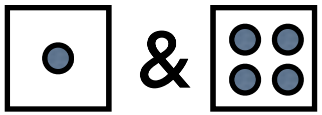
= 10

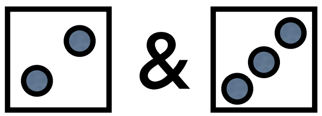
Back to the guessing game

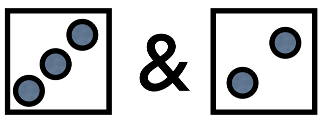
- Your friend rolls two fair dice, you have to guess what sum will turn up
- What are the possibilities?
- 2, 3, 4, 5, 6, 7, 8, 9, 10, 11,12
- Are all outcomes 2 to 12 equally likely?
- Let us do an experiment to find out


possible
outcomes
2
3
4
5
6
7
8
9
10
11
12

Back to the guessing game

 = 5

 = 5

 = 5

Is that it?
Or are there more ways we can get a 5?

Let us count all outcomes

← second die →

↑ first die ↓

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

How often does each outcome show up?

← second die →

↑ first die ↓

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

outcome	ways
2	**1**
3	**2**
4	**3**
5	**4**
6	**5**
7	**6**
8	**5**
9	**4**
10	**3**
11	**2**
12	**1**
	36

So what is the probability?

- How often would 7 appear?

  6 out of 36 times

- What is the probability of 7 appearing?

  6/36 = 0.1667

- What is the probability of 4 appearing?

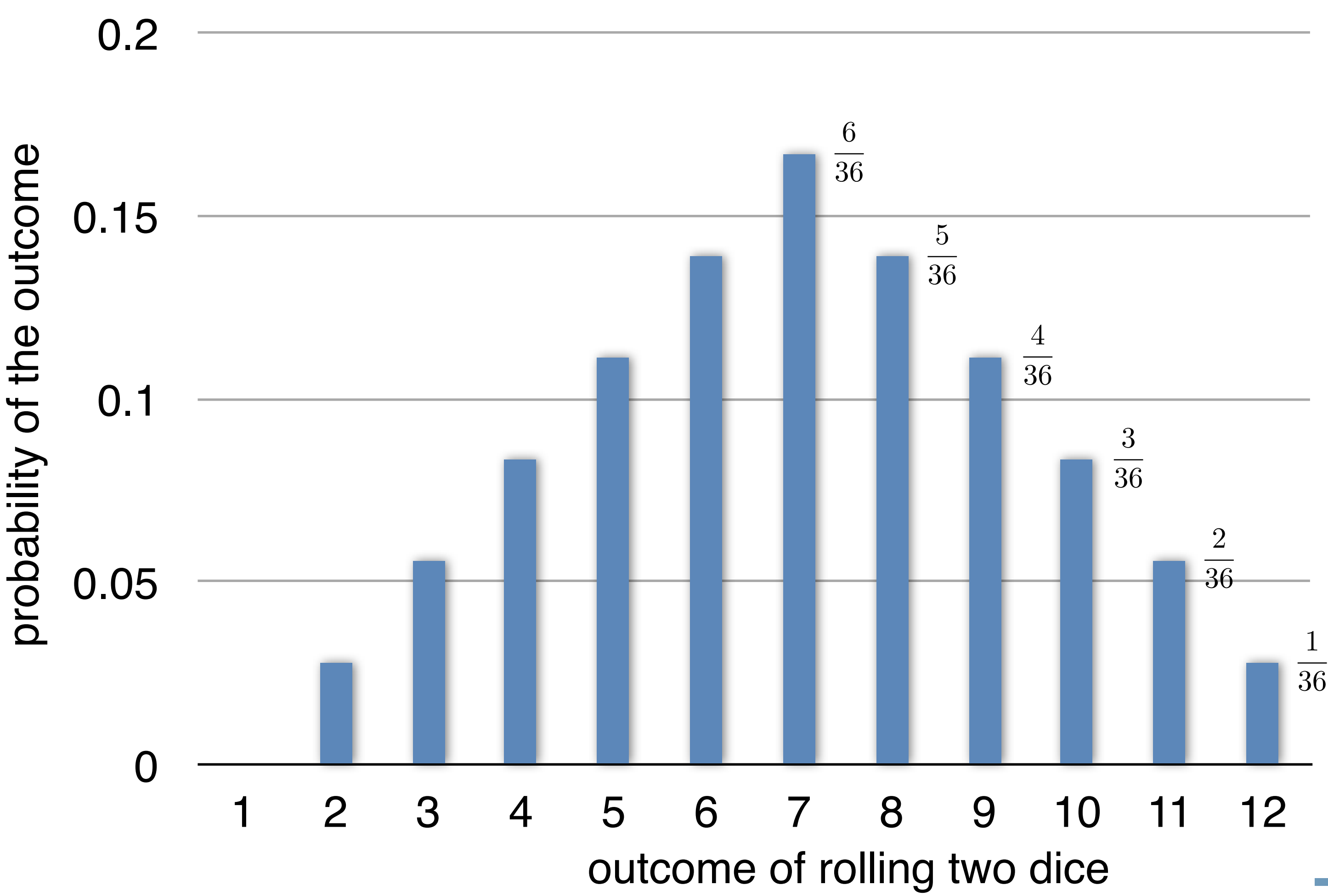
  3/36 = 0.0833

outcome	ways
2	**1**
3	**2**
4	**3**
5	**4**
6	**5**
7	**6**
8	**5**
9	**4**
10	**3**
11	**2**
12	**1**
	36

So what is the probability?

outcome	ways	probability
2	1	1/36 = 0.027778
3	2	2/36 = 0.055556
4	3	3/36 = 0.083333
5	4	4/36 = 0.111111
6	5	5/36 = 0.138889
7	6	6/36 = 0.166667
8	5	5/36 = 0.138889
9	4	4/36 = 0.111111
10	3	3/36 = 0.083333
11	2	2/36 = 0.055556
12	1	1/36 = 0.027778

Graphically... what is the probability?



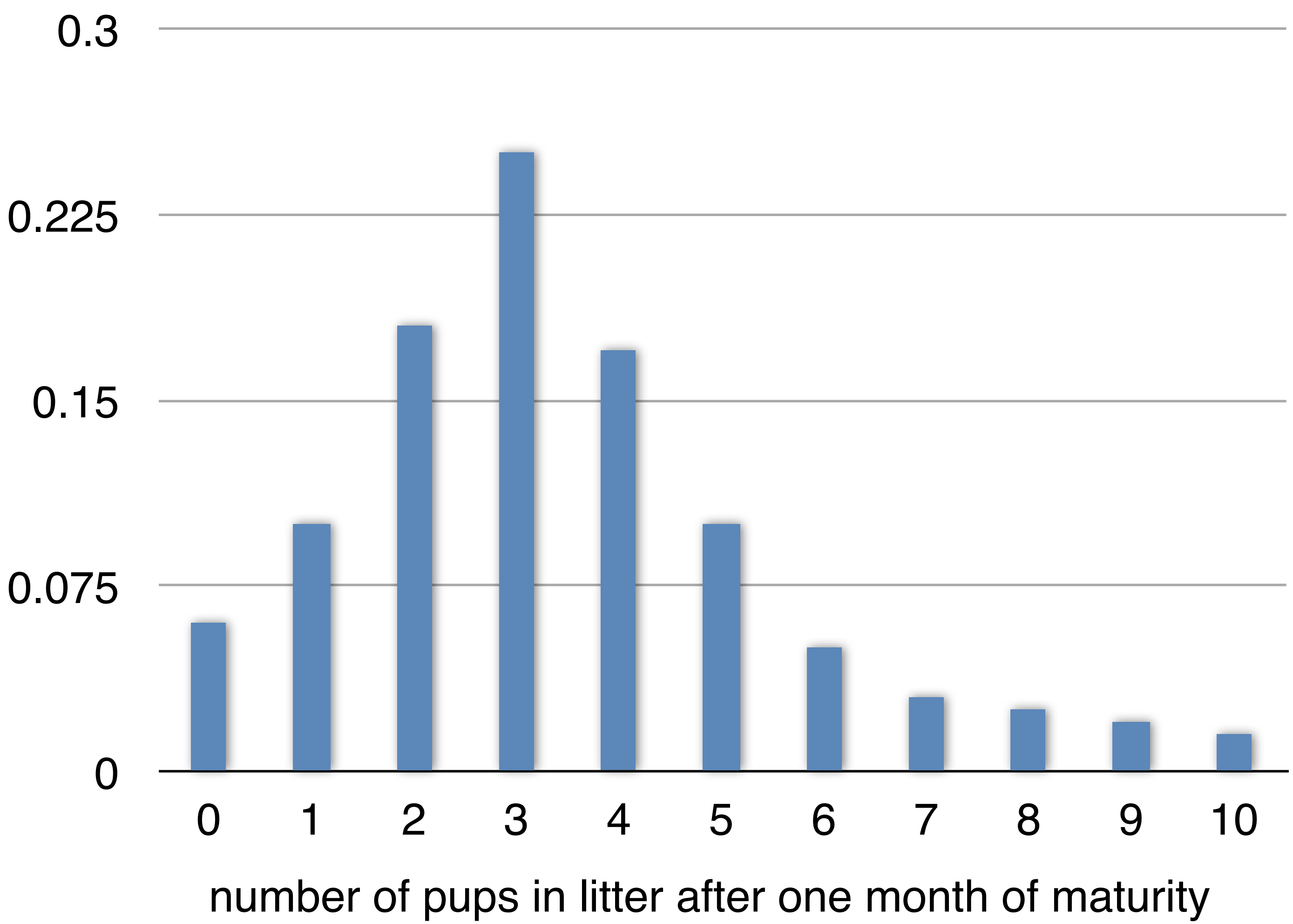
You got a different *distribution* of the outcomes by using two dice.

See what you get when you use three dice (at home)

Back to the rabbits

- Incorporate probabilities
  - birth
  - gender
  - weather/natural calamities
- growth of carrots
- multiple populations
  - competitors and predators
- Assumptions are key

Probability distribution for number of pups

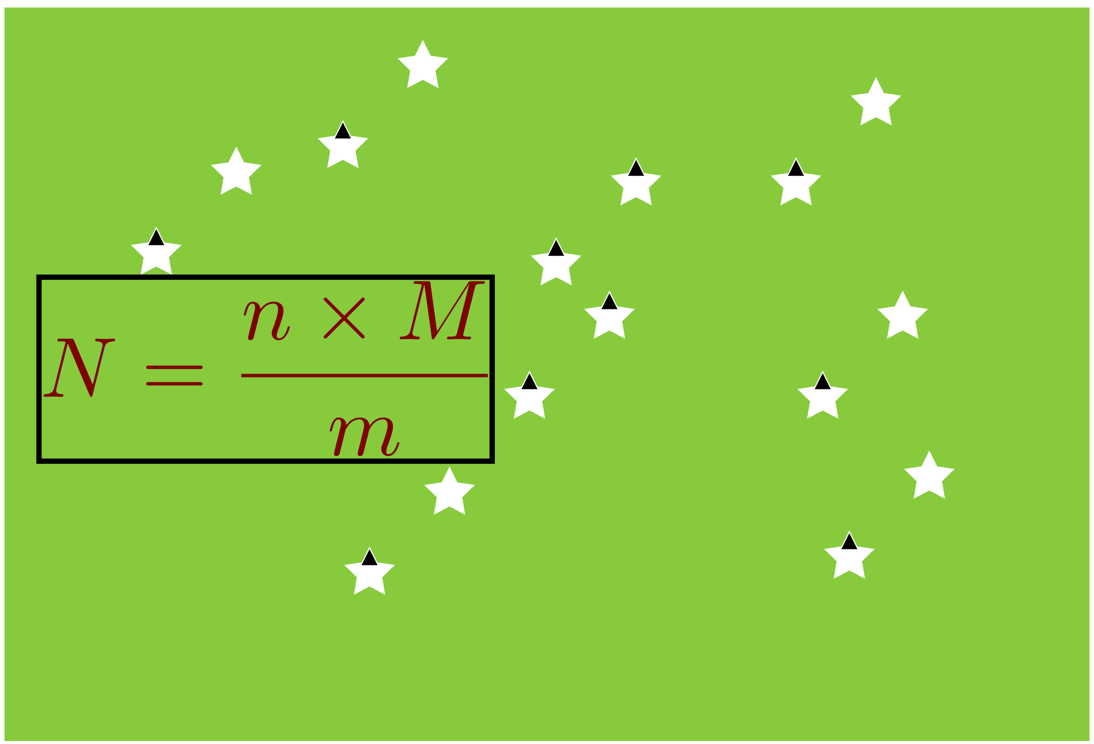


How do we know if our model is right?

- How can we “count” the number of rabbits?
- Method called “mark and recapture”

Mark and recapture

1. Capture a sample of *M* rabbits
2. Mark the *M* rabbits
3. Send them back in the field
4. Capture another sample of *n* rabbits



$$\boxed{N = \frac{n \times M}{m}}$$

$$M : N$$

✗ N total number of rabbits

✔ M number of marked rabbits

✔ n number of rabbits captured 2nd time

✔ m number of marked rabbits in 2nd set

Jackdaw with a ring

source: wikipedia

Snail marked with a number

source: wikipedia

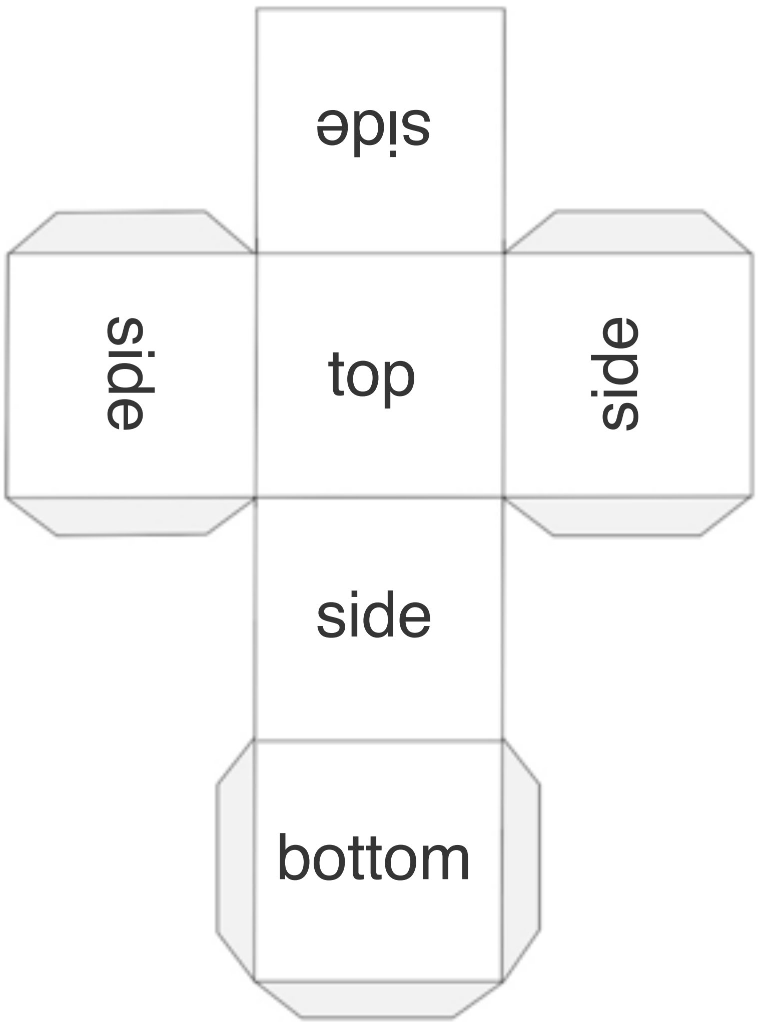
Mark and recapture

- Assumptions
  - Marks should not fall off!
  - No population growth/loss between the two captures
  - The animals don’t move out or move in to the field
  - A marked animal does not become easier or more difficult to catch the second time
  - Time between two samples is key

Why are we interested?

- Develop a model that will estimate the population of rabbits over a period of time
- How does it change with inputs?
- Not only for rabbits, but to predict forest cover, wildlife numbers etc.

Make your own ***loaded*** die



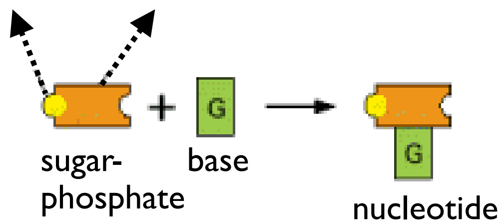
attach a small piece of cardboard

Math in molecular biology

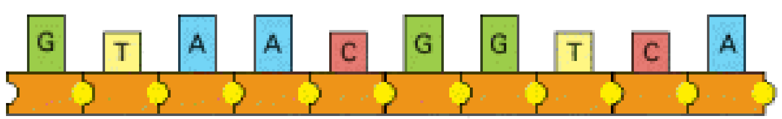
- Loads of mathematical modeling there!

DNA: code for life

Building block of DNA



Single strand of DNA



Four types of nucleotides:

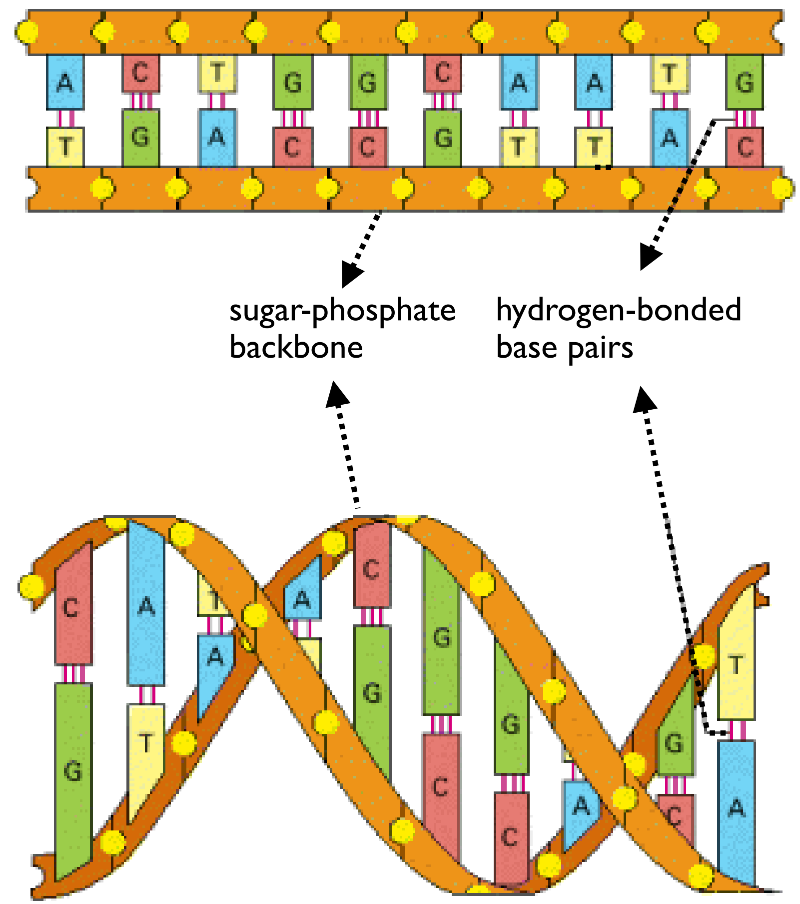
A: adenine

C: cytosine

G: guanine

T: thymine

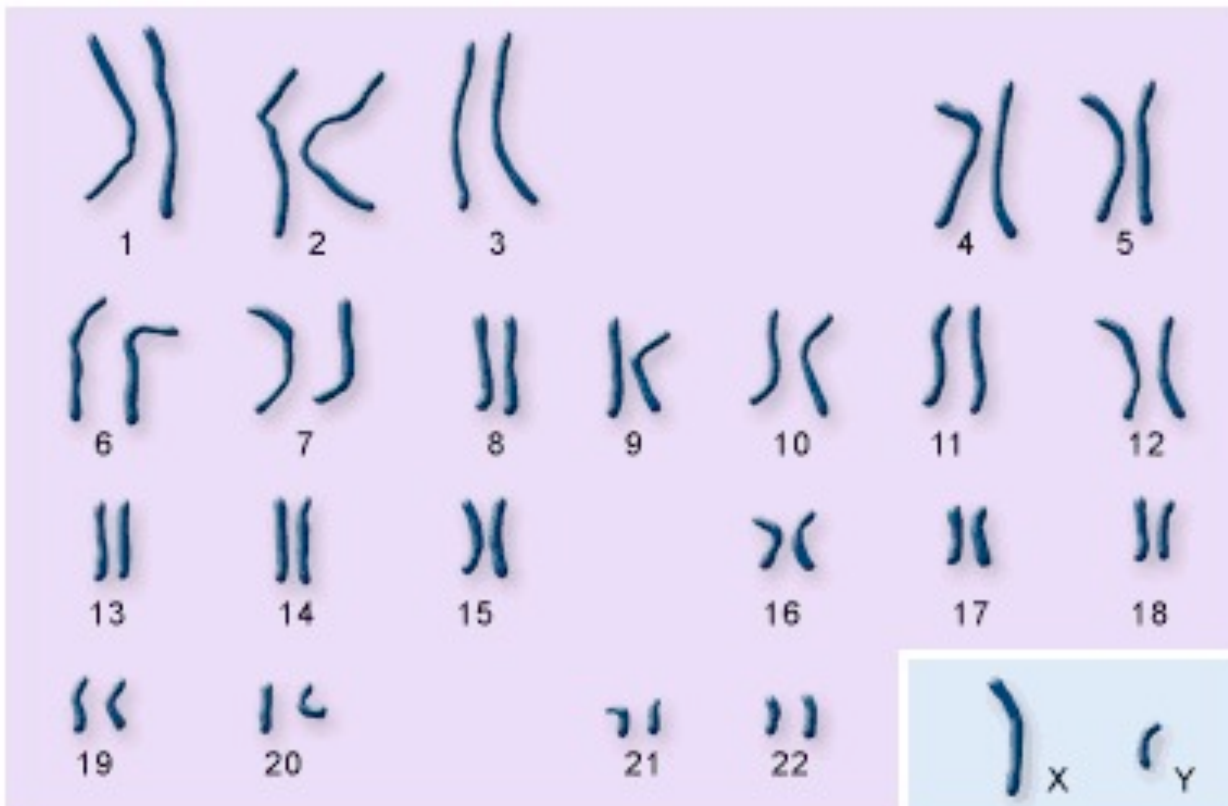
Double stranded DNA



Adapted from *Molecular Biology of the Cell*

The human genome

12,000,000,000 nucleotides!



ghr.nlm.nih.gov

1500 nucleotide region of chromosome 6

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...GGCTCCCCAGAACTGGCTGGGCCCCTGGGGACAGAGCCACCCCATGAGCTCGGGGTCCACCAGTGTG
TGGGGGAGATTCTGGGTTTGCCCAGTCCTGGGTTGTTTCCAGGAGAAAGCCGGGGGAGGGGCCCTCAGGC
CATTCCCCAACGGGGTGGGGAGGGTGACCCACAGCTCTGGGCCTCTTTTTGCCCTTTAGGGCTGTTGCTA
GGGAGAGGGAAGAGGGAGACCAAATGTCGGGGTTGGGGTGGGAGGGCGTCAGGCAGAGGCAACTGACTTC
ATTTGTGCCACACGCATGGGCATTGCAGCCTTGCGCTGTCCCAGGCATGCAGCTGCCTGGGGCCCAAGTT
GCAGTGAGCAGGGTGGGGTCTGGGAGGGGGTGAGAGGCAGGAATGGGGGTCAGAAGAAGTGGGAGCAGCT
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