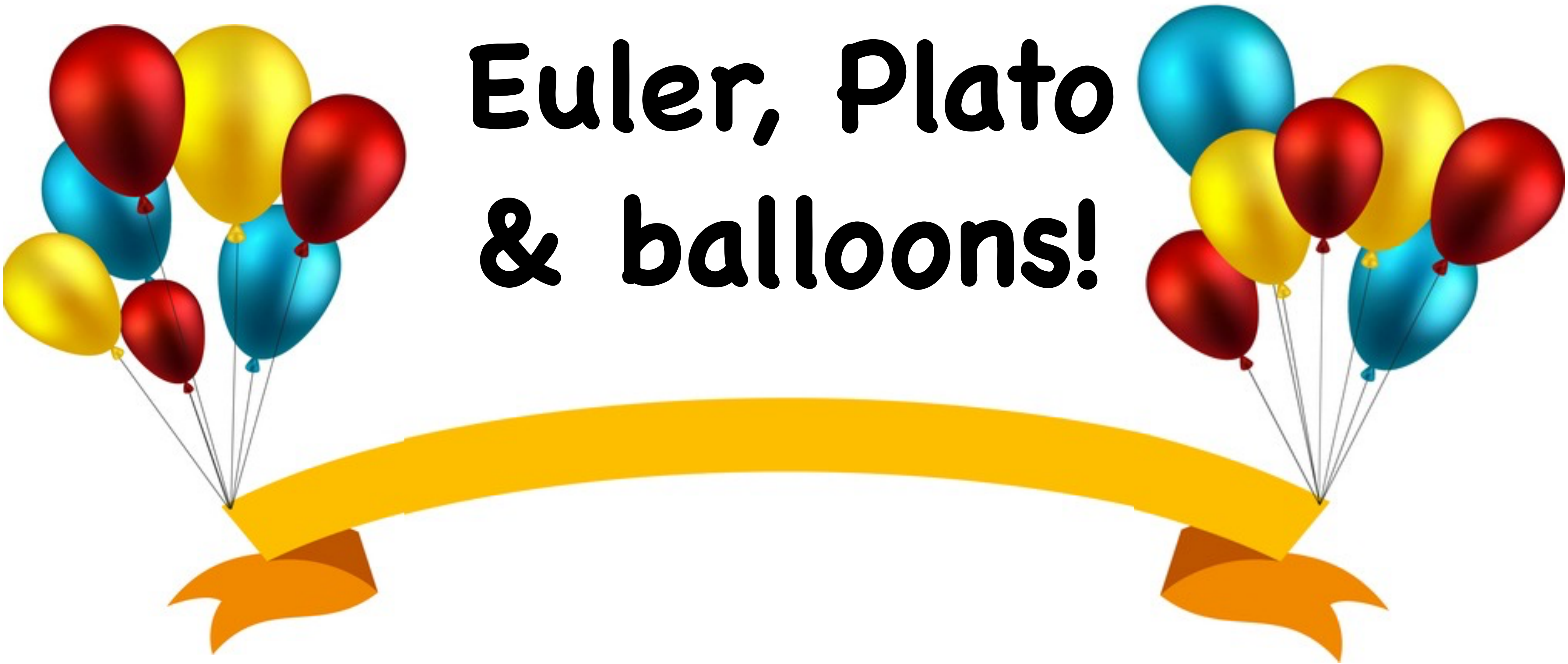
Euler, Plato & balloons!



Euler: The master of us all!

- Born in Basel, Switzerland in 1707 to a pastor.
- Got his Master of Philosophy degree at the ripe young age of 16.
- Moved to St Petersburg, Russia to become Professor of Physics and soon the Chair of the Mathematics and Geography Department at age 26.
- Died while working out the orbit of the newly discovered planet of Uranus at age 76.

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- Euler's identity:

$$e^{i\pi} + 1 = 0$$

e = Σ1/(n!) = 1 + 1 + 1/2 + 1/(2x3) + 1/(2x3x4) + … = 2.71828… is called Euler's number.

π = Ratio of circumference of a circle to its diameter = 3.1415926…

i = $\sqrt{}$(-1)

Sheldon: The master of us all!

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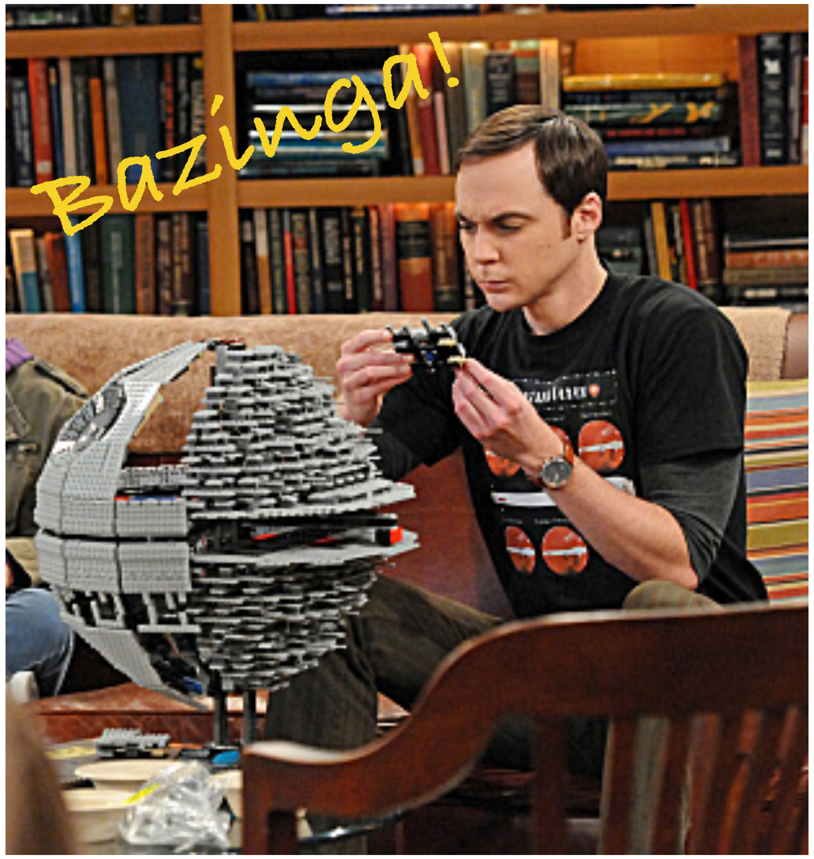

Euler: Designing death-stars

- Metal plates (faces) must be polygons.
- Windows along every edge.
- Lazer guns at every corner (vertices).

Euler: Designing death-stars

- Metal plates (faces) must be polygons.
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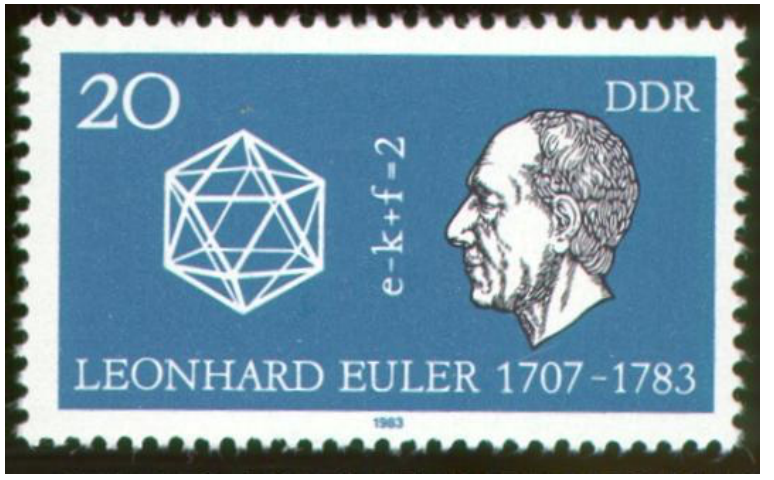
- Lazer guns at every corner (vertices).

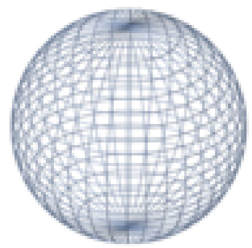
- *Design your own death-star on your balloon. Count and note the number of metal plates (faces), windows (edges) and guns (vertices).*

Euler Characteristic

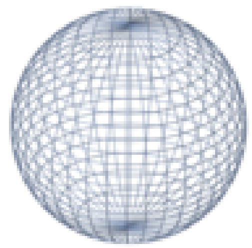
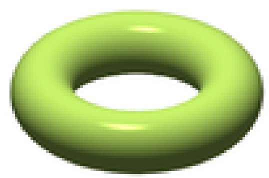
- *Calculate the Euler Characteristic of your balloon: Vertices - Edges + Faces*



Euler Characteristic

Sphere		**2**
Torus (Product of two circles)		**0**
Double torus		**−2**
Triple torus		**−4**

Euler Characteristic

Sphere		**2**
Torus (Product of two circles)		**0**
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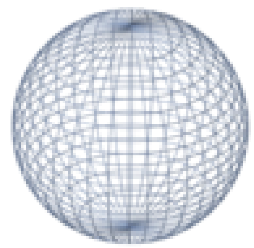
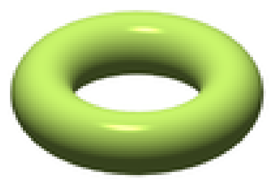
- Euler characteristic does not depend on the tiling of the surface

Euler Characteristic

Sphere		**2**
Torus (Product of two circles)		**0**
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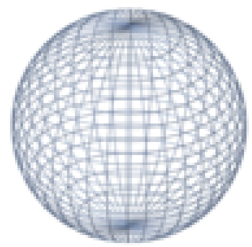
Euler Characteristic

Sphere		**2**
Torus (Product of two circles)		**0**
Double torus		**–2**
Triple torus		**–4**

- Euler characteristic does not depend on the tiling of the surface or deformations of the surface but it does depend on the overall shape of the surface.

Platonic Solids

A platonic solid is a regular convex polyhedron:

- Every face has same number of edges (i.e., all triangles, all squares, all pentagons etc).
- Every face is a regular polygon, i.e, each face has all edges of equal length and all corners (vertices) have the same angle.
- Every corner has same number of edges on it.

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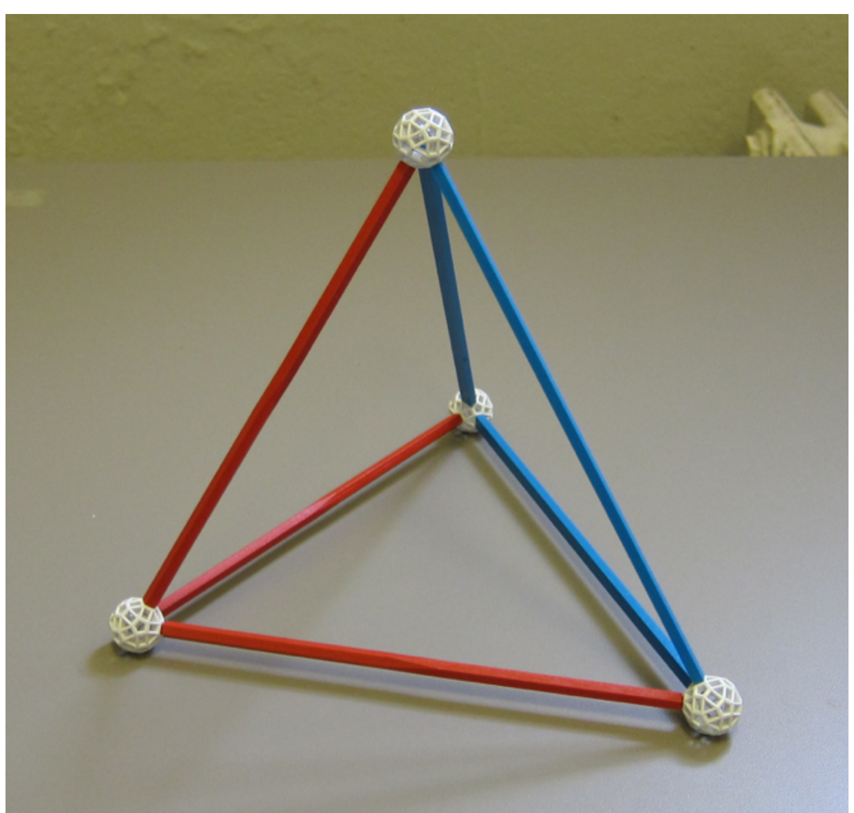

Tetrahedron

V=4
E=6
F=4
d=3 (No. of edges at a corner)
n=3 (No. of edges on each face)

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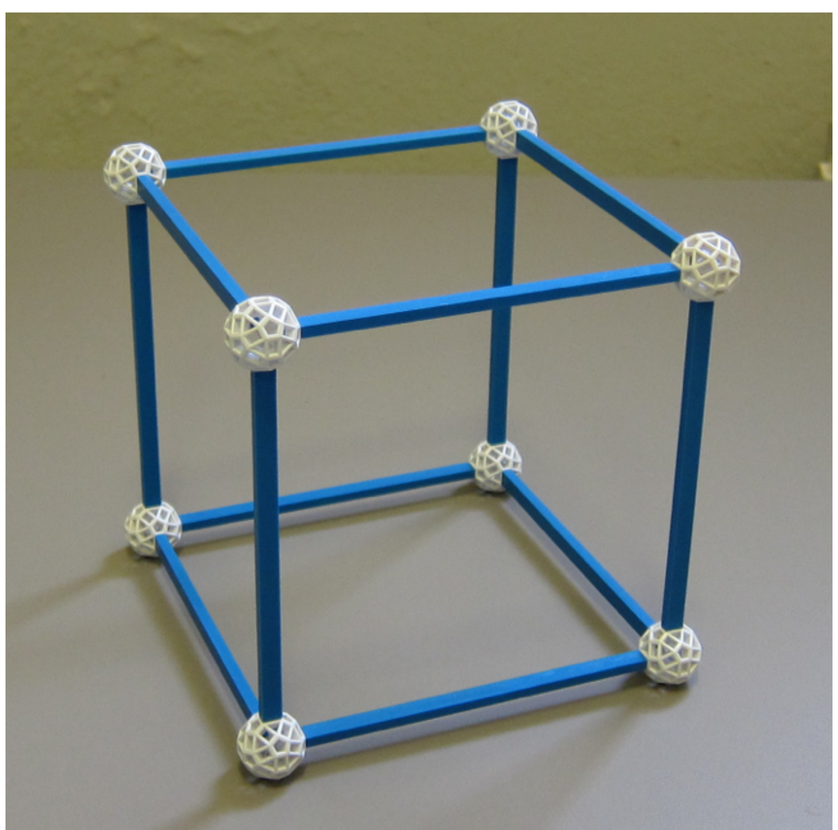

Cube (Hexahedron)

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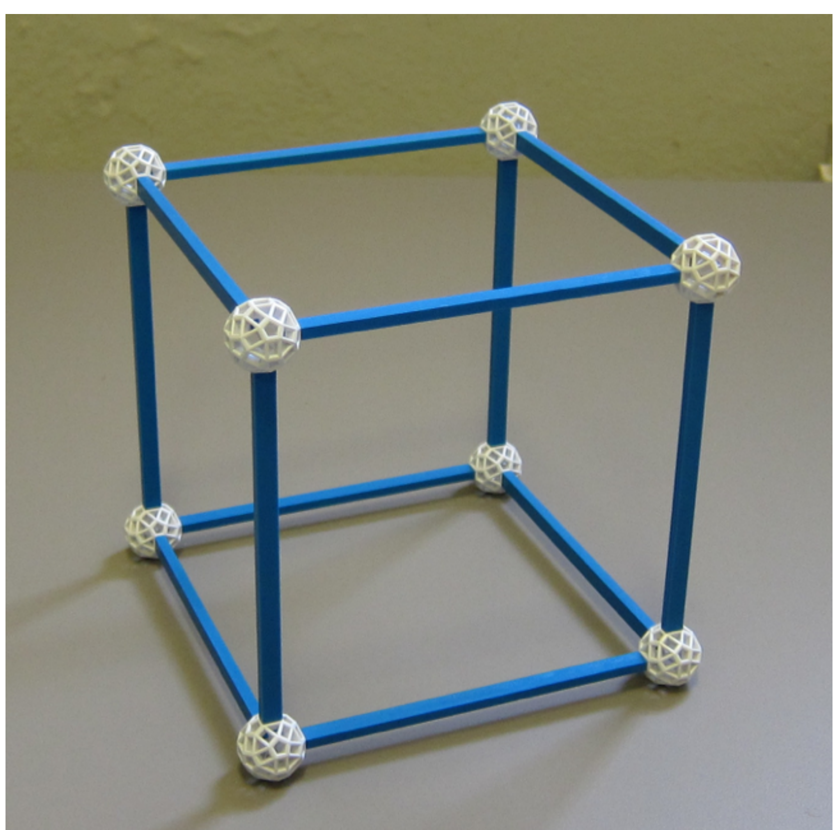

Cube (Hexahedron)

V=8
E=12
F=6
d=3 (No. of edges at a corner)
n=4 (No. of edges on each face)

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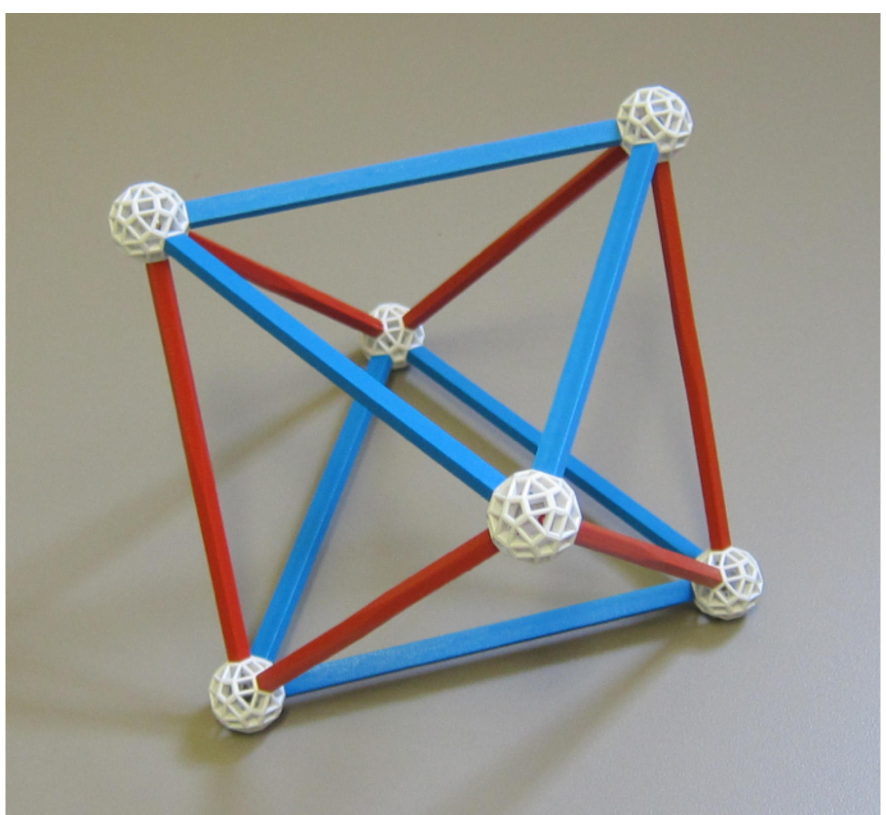

Octahedron

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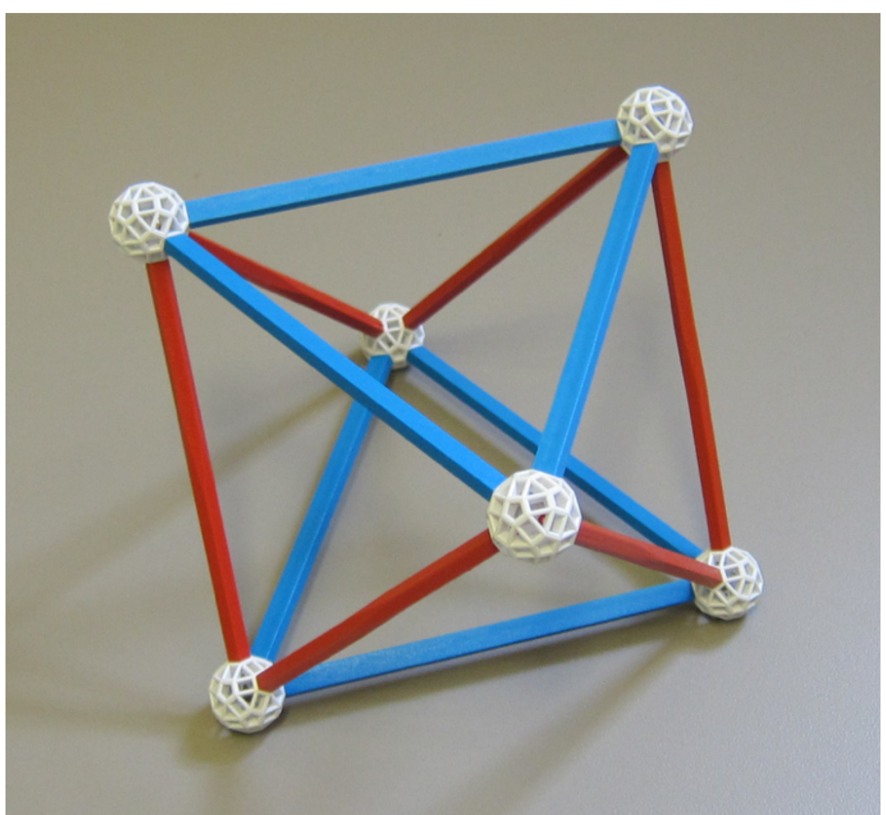

<u>Octahedron</u>

V=6
E=12
F=8
d=4 (No. of edges at a corner)
n=3 (No. of edges on each face)

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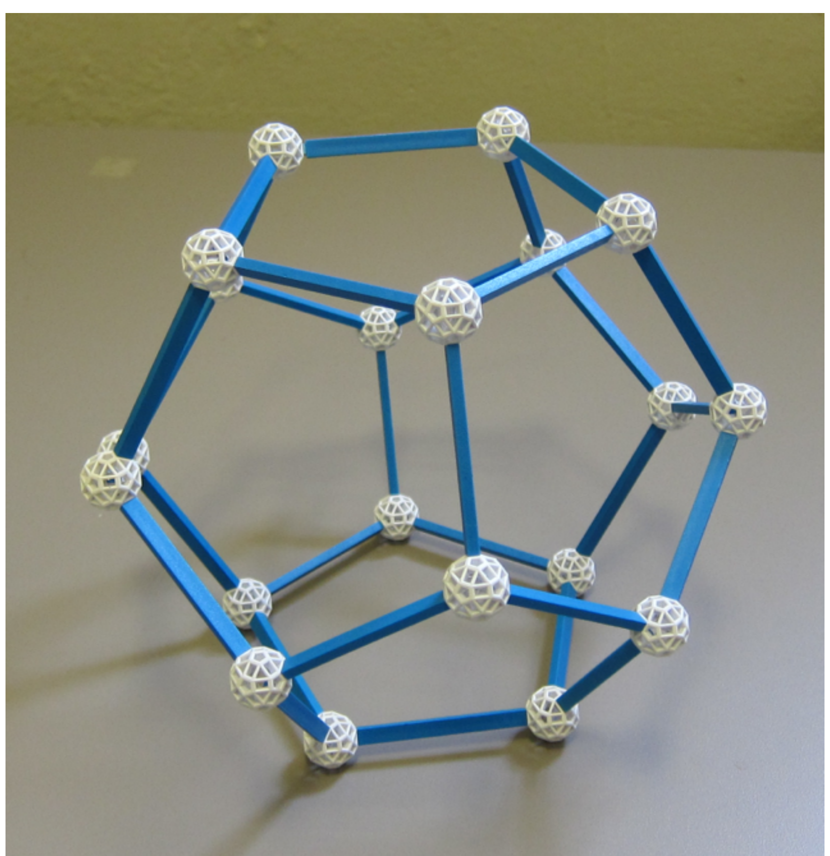

Dodecahedron

V=20
E=30
F=12
d=3 (No. of edges at a corner)
n=5 (No. of edges on each face)

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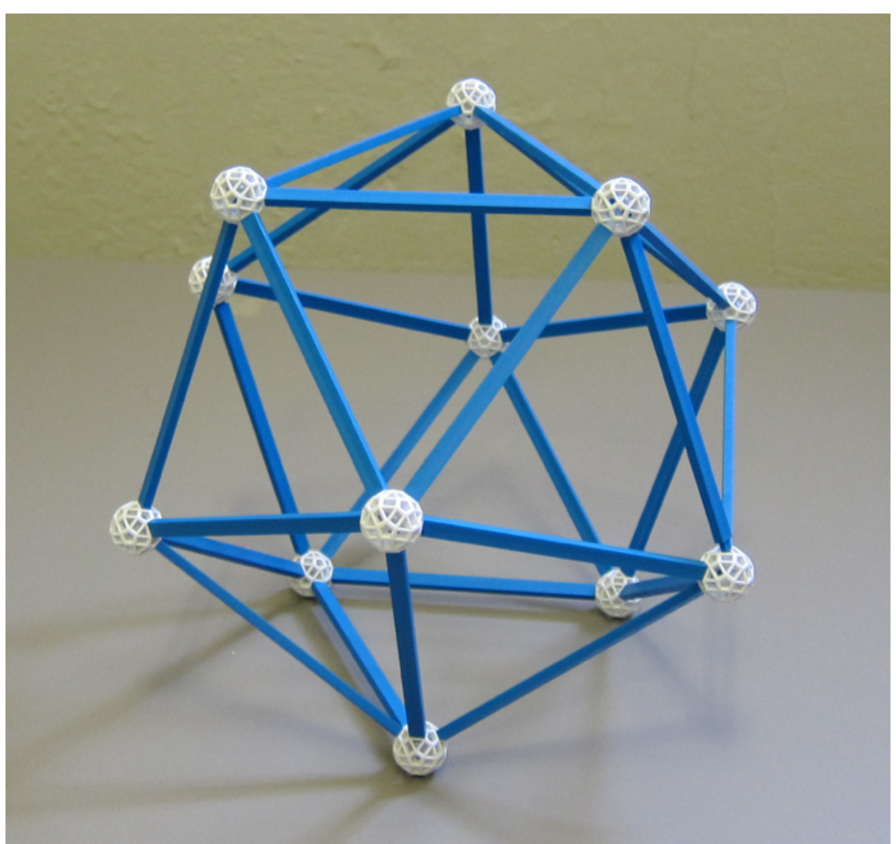

<u>Icosahedron</u>

V=12
E=30
F=20
d=5 (No. of edges at a corner)
n=3 (No. of edges on each face)

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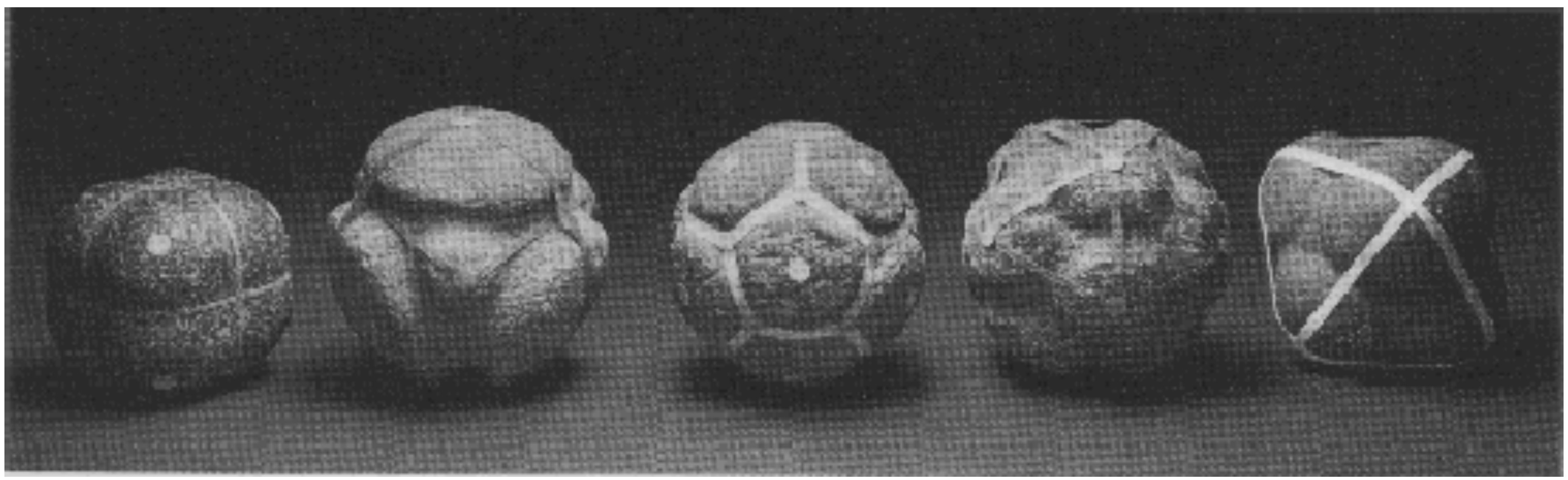
<u>What-else-ahedron?</u>

Plato

- Plato was a Classical Greek philosopher (approx 428-328 BC) and founder of the Academy in Athens, the first institution of higher learning in the Western world.

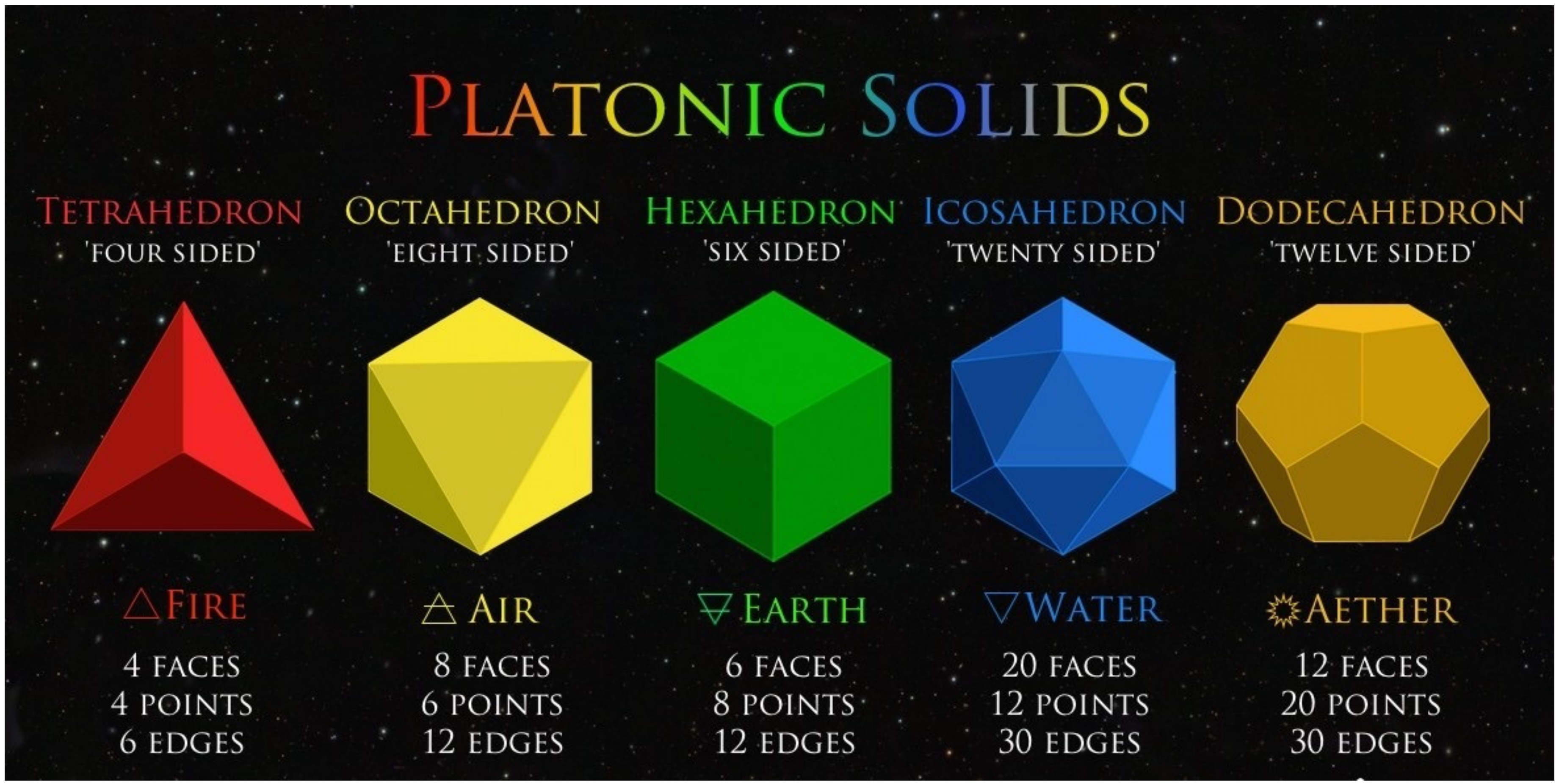
- Along with his teacher, Socrates, and his most famous student, Aristotle, Plato laid the very foundations of Western philosophy and science.

Platonic Solids through history



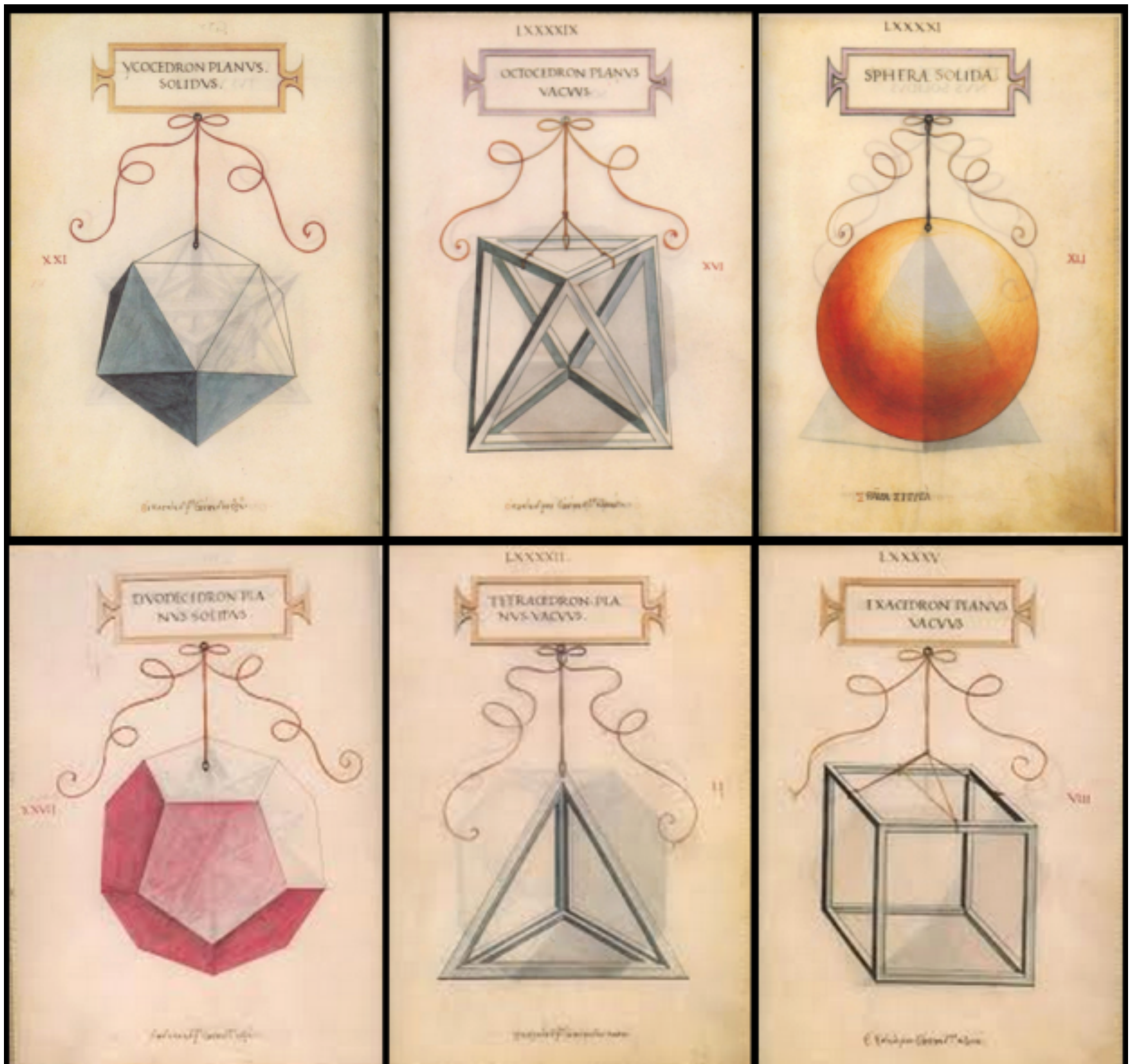
Neolithic stone sculptures found in Scotland
from a thousand years before Plato.

Platonic Solids through history



The Platonic solids are prominent in the philosophy of Plato. He wrote about them in the dialogue Timaeus 360 BC in which he associated each of the five classical elements (fire, air, earth, water, ether) with regular solids.

Platonic Solids through history



Leonardo Da Vinci (1452-1519) an Italian Renaissance genius and one of the greatest painters of all time illustrated the mathematics book 'De divina proportione' by Luca Pacioli.

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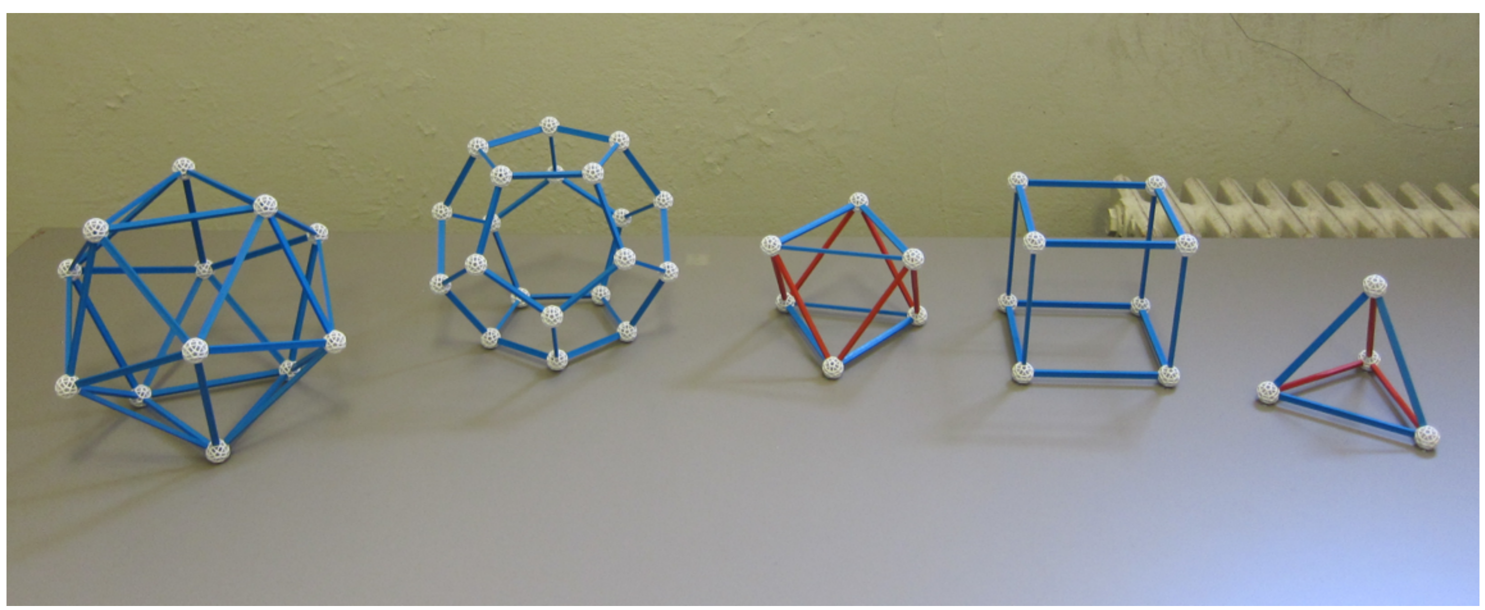

Are these all the Platonic solids?

Platonic Solids

- Suppose we have a Platonic solid with V vertices, E edges, F faces, n edges on each face and d edges on each vertex.

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- Then we have the following relations:

1. $V - E + F = 2$

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Name	Image	Vertices V	Edges E	Faces F	Euler characteristic: $V - E + F$
Tetrahedron		4	6	4	2
Hexahedron or cube		8	12	6	2
Octahedron		6	12	8	2
Dodecahedron		20	30	12	2
Icosahedron		12	30	20	2

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- Suppose we have a Platonic solid with V vertices, E edges, F faces, n edges on each face and d edges on each vertex.
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2. $nF=2E$

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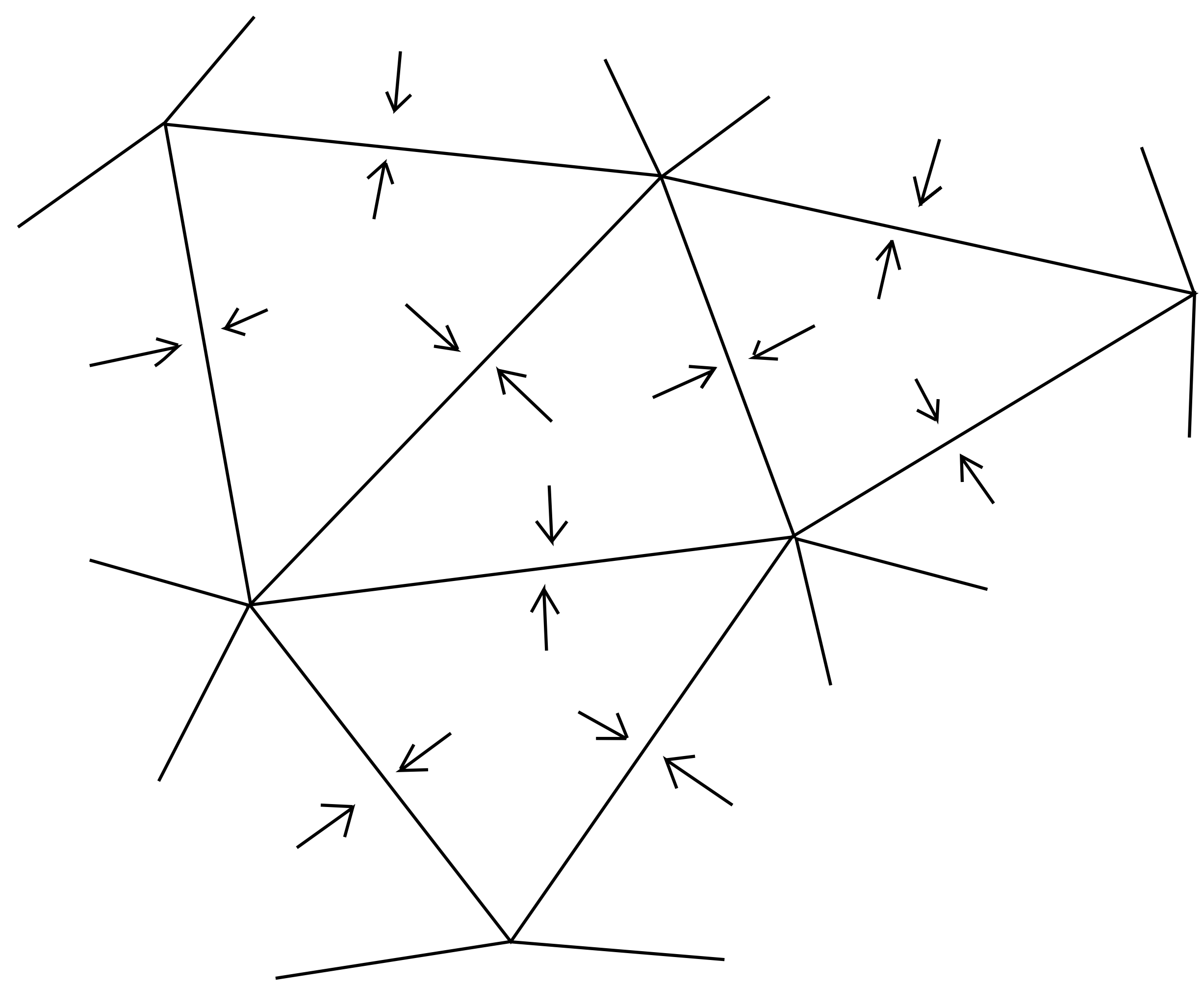

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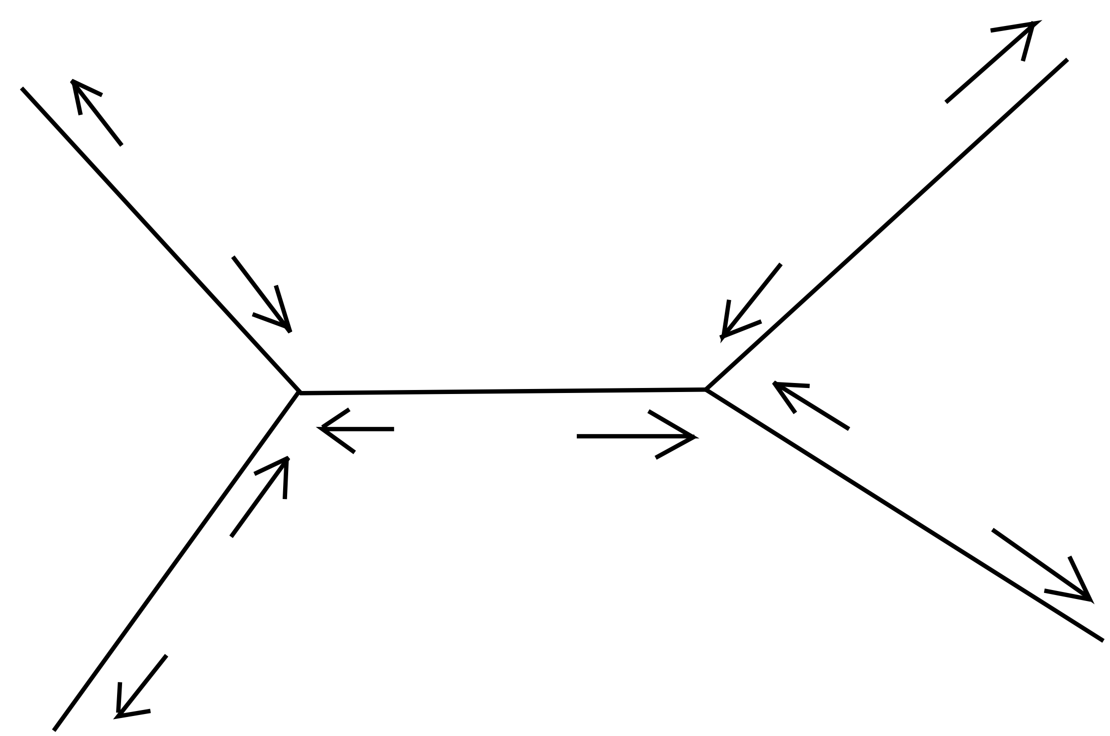

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- Suppose we have a Platonic solid with V vertices, E edges, F faces, n edges on each face and d edges on each vertex.
- Then we have the following relations:

1. $V - E + F = 2$
2. $nF=2E$
3. $dV=2E$
4. $n>2$
5. $d>2$

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- Combining these we get:

$2E/d - E + 2E/n = 2$

$1/d - 1/2 + 1/n = 1/E > 0$

$1/d + 1/n > 1/2$

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- *What are the values for n and d that satisfy this relation?*

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- Combining these we get:

$2E/d - E + 2E/n = 2$

$1/d - 1/2 + 1/n = 1/E > 0$

$1/d + 1/n > 1/2 = 0.5$

Decimal expansions:

$1/2=0.5$

$1/3=0.33$

$1/4=0.25$

$1/5=0.2$

$1/6=0.17$

$1/7=0.14$

$1/8=0.13$

$1/9=0.11$

$1/10=0.1$

- *What are the values for n and d that satisfy this relation?*

Platonic Solids

- The relation $1/d + 1/n > 1/2$ gives us 5 possible values for (n,d) which are:

1. $n=3, d=3$
2. $n=4, d=3$
3. $n=3, d=4$
4. $n=5, d=3$
5. $n=3, d=5$

Platonic Solids

- The relation $1/d + 1/n > 1/2$ gives us 5 possible values for (n,d) which are:

1. $n=3, d=3$
2. $n=4, d=3$
3. $n=3, d=4$
4. $n=5, d=3$
5. $n=3, d=5$

- *Now using $nF=2E$, $dV=2E$ and $V - E + F = 2$ find the values for V, E and F.*

Platonic Solids

- The relation $1/d + 1/n > 1/2$ gives us 5 possible values for (n,d) and using the other relations $nF=dV=2E$ and $V-E+F=2$:

1. $n=3$, $d=3$, $V=4$, $E=6$, $F=4$
2. $n=4$, $d=3$, $V=8$, $E=12$, $F=6$
3. $n=3$, $d=4$, $V=6$, $E=12$, $F=8$
4. $n=5$, $d=3$, $V=20$, $E=30$, $F=12$
5. $n=3$, $d=5$, $V=12$, $E=30$, $F=20$

- *Now using $nF=2E$, $dV=2E$ and $V - E + F = 2$ find the values for V, E and F.*

Platonic Solids

- The relation $1/d + 1/n > 1/2$ gives us 5 possible values for (n,d) and using the other relations $nF=dV=2E$ and $V-E+F=2$:

1. $n=3, d=3, V=4, E=6, F=4$ Tetrahedron
2. $n=4, d=3, V=8, E=12, F=6$ Cube
3. $n=3, d=4, V=6, E=12, F=8$ Octahedron
4. $n=5, d=3, V=20, E=30, F=12$ Dodecahedron
5. $n=3, d=5, V=12, E=30, F=20$ Icosahedron

- *So, using the idea of Euler Characteristic we have shown that these are the only possible Platonic Solids!*

Commemorating Euler

1950: German stamp for 250th anniversary of Berlin Academy of Science

Commemorating Euler

1957: German stamp for 250th birth anniversary

Commemorating Euler

1957: Russian stamp for 250th birth anniversary

Commemorating Euler

1957: Swiss stamp for 250th birth anniversary

Commemorating Euler

1979: Swiss Frank banknote

Commemorating Euler

1983: German stamp for 200th death anniversary

Comm uler

2007: Swiss stamp for 300th birth anniversary

Com uler

2007: Russian Rouble for 300th birth anniversary

Comp

2009: Guinea Bissau stamp

Com

2013: Google Doodle

Comm

Thank you!